MAT 146, Spring 2019
Practice problems for the final exam

Note that this practice sheet contains more problems than the actual midterm

1. Let b_n be the number of ways to change n cents using coins valued 4 and 6 cents. Find the generating function $\sum b_n x^n$. For which n we can actually change n cents this way (that is, $b_n > 0$)?

2. A permutation f has order 3 if for all x one has $f(f(f(x))) = x$ and f is not the identity permutation. Find the exponential generating function for the number of permutations of order 3.

3. The generating function for the sequence a_n equals $\sum a_n x^n = A(x)$. Find the generating function for the sequence $b_n = 5a_{n+1} + a_{n-1} - 3$.

4. Consider the series

$$A(x) = \sum a_n x^n = \frac{(1+x)(1+2x)}{(1-2x)(1+x^2)}.$$

(a) Find the poles of $A(x)$ and principal parts at these poles.

(b) Find the radius of convergence for $A(x)$.

(c) Use (b) to estimate the coefficients a_n.

(d) Use (a) to find the partial fraction decomposition of $A(x)$

(e) Find the closed formula for a_n.

5. The sequence a_n satisfies the recurrence relation $a_n = a_{n-1} + 2a_{n-2}, a_0 = a_1 = 1$. Find the generating function $\sum a_n x^n$.

6. The exponential generating function $A(x) = \sum \frac{a_n}{n!} x^n$ satisfies the differential equation

$$A'(x) + A(x) = e^x,\ A(0) = 0.$$

(a) Find the closed formula for the coefficients a_n.

(b) Find the closed formula for the series $A(x)$.

7. Find the closed formula for the coefficients of the series:
(a) $A(x) = \frac{1+x}{(1+2x)(1+3x)}$.

(b) $A(x) = \frac{1+x}{(1-x)^3}$.

8. Find the generating function for the number of partitions **without** parts equal to 3 or 5.

9. Use the exponential formula to find the number of set partitions **without** blocks of size 3 or 5.

10*. The *Bessel function* $A(x) = \sum_{n=0}^{\infty} a_n x^n$ satisfies the differential equation

$$xA''(x) + A'(x) + xA(x) = 0.$$

Find a closed formula for its coefficients.