MAT 146, Spring 2019
Homework Assignment 4

Due before the start of the class on Monday, May 6

Section 1.7: 4. Let $f(x)$ be the **exponential** generating function of a sequence $\{a_n\}$. Find the exponential generating functions for the following sequences in terms of $f(x)$: (a) $\{a_n + c\}$; (b) $\{\alpha a_n + c\}$ (c) $\{na_n\}$; (e) $0, a_1, a_2, a_3, \ldots$; (g) $a_0, 0, a_2, 0, a_4, 0, \ldots$; (h) $a_1, a_2, a_3, \ldots$
8. (a) A sequence $\{a_n\}$ satisfies the recurrence relation $a_{n+1} = 3a_n + 2,\ a_0 = 0$. Find the exponential generating function $\sum_{n=0}^{\infty} \frac{a_n}{n!} x^n$.
Section 2.7: 20. Prove the binomial theorem

$$(x+y)^n = \sum_k \binom{n}{k} x^k y^{n-k}$$

by comparing the coefficient of $t^n/n!$ on both sides of the equation $e^{t(x+y)} = e^{tx}e^{ty}$. Prove the multinomial theorem

$$(x_1 + \ldots + x_k)^n = \sum_{r_1 + \ldots + r_k = n} \frac{n!}{r_1! \cdots r_k!} x_1^{r_1} \cdots x_k^{r_k}$$

by a similar method.
27. Let $D(n)$ be the number of derangements on n letters. We proved in class that the exponential generating function for $D(n)$ has the form

$$D(x) = \frac{e^{-x}}{1-x} = \sum_n \frac{D(n)}{n!} x^n.$$

(b) Prove, by any method, that $D(n+1) = (n+1)D(n) + (-1)^{n+1}$
(c) Prove, by any method, that $D(n+1) = n(D(n) + D(n-1))$
(e) Let $D_k(n)$ be the number of permutations of n letters with exactly k fixed points. Show that

$$\sum_{k,n} D_k(n) \frac{x^n y^k}{n!} = \frac{e^{-x(1-y)}}{1-x}.$$

The homework must be legible, and written in connected sentences that explains what you are doing. Just the answer (whether correct or not) is not enough. Please put your name and section number on every page and staple the pages together. Homework should be handed in on time, late homework will not be graded.