

MAT 146, Spring 2019
Homework Assignment 5

Due before the start of the class on Monday, May 13

Section 3.19: 2. Let $f(n, k)$ be the number of permutations of n letters whose cycle lengths are all divisible by k . Find a simple explicit expression for the exponential generating function $\sum_n f(n, k) \frac{x^n}{n!}$.

3. Find the exponential generating function for the number of set partitions of $\{1, \dots, n\}$ where all parts have prime number of elements.

5. Let T_n be the number of involutions in S_n , we proved in class that

$$T(x) = \sum_n \frac{T_n}{n!} x^n = e^{x+x^2/2}.$$

(a) Find a recurrence for T_n .

(b) Compute T_1, \dots, T_6 .

(c) Give a combinatorial interpretation of the recurrence.

6. Find the exponential generating function for the number of permutations in S_n with no cycles of length ≤ 3 . Your answer should not contain any infinite series.

The homework must be legible, and written in connected sentences that explains what you are doing. Just the answer (whether correct or not) is not enough. Please put your name and section number on every page and staple the pages together. Homework should be handed in on time, late homework will not be graded.