

MAT 146, Spring 2019  
Homework Assignment 7

**Due before the start of the class on Wednesday, June 5**

*Section 2.7:* 11. Find the radius of convergence of the following series:

(a)  $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$ ; (c)  $1 + 5x^2 + 25x^4 + 125x^6 + \dots$

**A:** (a) Find the coefficients of the power series

$$A(x) = \sum_{n=0}^{\infty} a_n x^n = \frac{1-x}{(1-2x)(1+3x)}.$$

(b) Find the radius of convergence of this series.

**B:** The sequence  $\{a_n\}$  satisfies the recurrence relation

$$15a_n - 5a_{n-1} + 3a_{n-2} - a_{n-3} = 0$$

with some initial conditions  $a_0, a_1, a_2$ .

(a) Find the radius of convergence of the series  $\sum a_n x^n$ .

(b) Estimate  $a_n$  without solving the recurrence.

**C:** Recall that the exponential generating function for the number of derangements equals

$$D(x) = \sum \frac{D_n}{n!} x^n = \frac{e^{-x}}{1-x}.$$

(a) Find all poles of  $D(x)$  and principal parts at these poles.

(b) Use “pole removal” procedure to estimate  $D_n$ .