MATH 148 Midterm 2

February 29, 2016

Name:____

ID:____

DO NOT OPEN THIS EXAM YET

(1) Fill in your name and ID.

(2) This exam is closed-book and closed-notes; no calculators, no phones.

(3) Please write legibly to receive credit. Circle or box your final answers. If your solution to a problem does not fit on the page on which the problem is stated, please indicate on that page where in the exam to find (the rest of) your solution.

(4) You may continue your solutions on additional sheets of paper provided by the proctor. If you do so, please write your name and ID at the top of each of them and staple them to the back of the exam (stapler available); otherwise, these sheets may get lost.

(5) Anything handed in will be graded; incorrect statements will be penalized even if they are in addition to complete and correct solutions. If you do not want something graded, please erase it or cross it out.

(6) Show your work; correct answers only will receive only partial credit (unless noted otherwise).

(7) Be careful to avoid making grievous errors that are subject to heavy penalties.

(8) If you need more blank paper, ask a proctor.

Out of fairness to others, please stop working and close the exam as soon as the time is called. A significant number of points will be taken off your exam score if you continue working after the time is called. You will be given a two-minute warning before the end.

1	2	3	4	5	Total

1. (25 points) The parity check matrix for a Hamming code has the following form:

$$G = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

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2. (25 points) Factor the polynomial $x^4 + x^2 + x + 1$ over $\mathbb{Z}_2$. Explain why the factors are irreducible.

3. (25 points) Simplify the following expressions in the field $\mathbb{Z}_2[x]/(x^2+x+1)$: (a) $1/x$ (b) x^{2016}.

4. (25 points) a) Write the generating matrix for the Reed-Muller code $G(1,4)$.

b) Find the minimal weight for this code.

c) How many errors does it correct?

This is a bonus problem. Please start this problem only if you completed the rest of the exam.

5*. (10 points) Let $p > 2$ be a prime number. Show that the polynomial $x^2 - d$ is irreducible over $\mathbb{Z}_p$ for exactly $(p-1)/2$ values of $d \in \mathbb{Z}_p$.