MAT 148, Winter 2016
Practice problems for Midterm 2

1. Find the generating matrices and the parity check matrices for the Reed-Muller codes $R(1,3)$ and $R(2,3)$.
2. Consider the Hamming code with the parity check matrix

$$H = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Decode the messages (a) 1001001 (b) 1000001 (c) 1110111.
3. Factor the polynomials over $\mathbb{Z}_2$: (a) $x^6 - 1$ (b) $x^8 - 1$ (c) $x^5 + x^3 + x + 1$ (d)* $x^8 - x$.
4. Consider the finite field $F = \mathbb{Z}_2[x]/(x^3 + x^2 + 1)$. Simplify the following expressions in F: (a) $(x^2 + 1)(x^2 + x + 1)$ (b) x^{100} (c) $1/x$ (d) $1/(x^2 + 1)$.
5. Construct a field with 25 elements.
6*. Let p be a prime number.
a) Prove that $(a + b)^p = a^p + b^p \mod p$.
b) Suppose that $\underbrace{1 + \ldots + 1}_{p} = 0$ in some finite field F. Prove that the function $\Phi(x) = x^p$ is a bijection from F to itself.
7. The double-error correcting BCH code for $GF(16)$ has a parity check matrix with 8 rows and 15 columns.
(a) Find the minimal weight for this code.
(b) Does it satisfy the sphere-packing bound?
(c) Does it satisfy the VG bound?