MAT 148, Winter 2016
Solutions to HW 2

12. (25 points) Give generator and parity check matrices for the binary code consisting of all even weight vectors of length 8.

Solution: A vector has even weight if and only if the sum of its component is even, that is, equals 0 mod 2. Therefore the parity check matrix for this code equals

$$H = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} = (A^t | I_1).$$

Now the generator matrix equals

$$G = (I_7|A) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

14. (25 points) Give a parity check matrix for the [7,4] Hamming code with the generator matrix:

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Solution: If $G = (I_4|A)$ then

$$H = (A^t|I_3) = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}.$$

16. (25 points) Show that in a binary code either all the vectors have even weight or half have even weight and half have odd weight.

Solution: Suppose that one of the vectors w has odd weight. If x has even weight then $x + w$ has odd weight, and if x has odd weight then $x + w$ has even weight. Therefore the function $f(x) = x + w$ defines a bijection between the sets of vectors of even and of odd weight, and hence these sets must have the same cardinality.

23. (25 points) Is it possible to find 8 binary vectors of length 6 so that $d(u, v) \geq 3$ for any two of them? If so, can the eight vectors form a linear code?

Solution: Let us look for a linear code satisfying this property. Since $8 = 2^3$, it should be a [6,3] code, and all codewords should have weight at least 3, so each row of the 3×6 generator matrix should have at least 3 1's. One can therefore guess the generator matrix:

$$G = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}.$$

The list of all 8 codewords (all possible sums of rows of G) has the following form:

$$000000, 100110, 010101, 001011, 110011, 101101, 011110, 111000.$$

Since all nonzero codewords have weight at least 3, we get

$$d(u, v) = wt(u - v) \geq 3.$$

Note that since the code is linear, if u and v are codewords then $u - v$ is also a codeword.