

# MAT 148, Winter 2016 Homework Assignment 6

**Due before the start of the class on Wednesday, February 24**

Please read the sections 3.1-3.4 of the textbook before starting on the problem set.

**Written Assignment** (see p. 48-49):

- D.** Consider a field  $F$  with  $2^n$  elements. It is known that  $1 + 1 = 0$  in  $F$ .
- a) Prove that if  $a^2 = b^2$  then  $a = b$  ( $a, b \in F$ ). *Hint: factor  $a^2 - b^2$ .*
  - b) Prove that the function  $\Phi(x) = x^2$  is a bijection from  $F$  to itself.
  - c) Prove that every element in  $F$  has a unique square root.
  - d) Prove that  $\Phi(x + y) = \Phi(x) + \Phi(y)$  and  $\Phi(x)\Phi(y) = \Phi(x)\Phi(y)$  for all  $x, y \in F$ .
  - e) Describe  $\Phi$  explicitly for  $F = GF(16)$ .
12. Perform the following computations in  $GF(16)$  (field with 16 elements).
- (a)  $1001 \cdot 1011 + 0101/1100$ ; (b)  $\sqrt{1110} + 1101$ ; (c)  $\sqrt{1000}$ .
- 15a. Using the double-error-correcting BCH code, decode the vectors
- a)  $(0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$
  - b)  $(1, 1, 1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$

*The homework must be legible, and written in connected sentences that explains what you are doing. Just the answer (whether correct or not) is not enough. Please put your name and section number on every page and staple the pages together. Homework should be handed in on time, late homework will not be graded.*