MAT 150A, Fall 2015
Practice problems for Midterm 2

1. Prove that every group with 4 elements is isomorphic either to $\mathbb{Z}_4$ or to $\mathbb{Z}_2 \times \mathbb{Z}_2$ (and is therefore commutative).
2. Are the following matrices orthogonal? Do they preserve orientation? Describe the corresponding transformations geometrically.

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

3. Find the area of the parallelogram with vertices (0,0), (2,3), (3,2) and (5,5).
4. Let A be the matrix of reflection in $x - y$ plane, and let B be the matrix of reflection in $y - z$ plane.
a) Write the 3×3 matrices A and B explicitly.
b) Compute the matrix products AB and BA .
c) Prove that AB and BA are rotations and find the axis of these rotations.
5. Describe all homomorphisms (a) From $\mathbb{Z}_5$ to $\mathbb{Z}_7$ (b) From $\mathbb{Z}_4$ to $\mathbb{Z}_6$
6. Prove that the groups $\mathbb{Z}_6$ and S_3 are not isomorphic
7. If we label the diagonals in the square by 1 and 2, every isometry of the square would permute them. This gives a homomorphism from D_4 to S_2. Describe its kernel and image.