## MAT 150A, Fall 2017 Practice problems for Midterm 2

1. Prove that every group with 4 elements is isomorphic either to  $\mathbb{Z}_4$  or to  $\mathbb{Z}_2 \times \mathbb{Z}_2$  (and is therefore commutative).

2. Are the following matrices orthogonal? Do they preserve orientation? Describe the corresponding transformations geometrically.

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

3. Let A be the matrix of reflection in x - y plane, and let B be the matrix of reflection in y - z plane.

a) Write the  $3 \times 3$  matrices A and B explicitly.

b) Compute the matrix products AB and BA.

c) Prove that AB and BA are rotations and find the axis of these rotations.

4. Describe all homomorphisms (a) From  $\mathbb{Z}_5$  to  $\mathbb{Z}_7$  (b) From  $\mathbb{Z}_4$  to  $\mathbb{Z}_6$ 

5. Prove that the groups  $\mathbb{Z}_6$  and  $S_3$  are not isomorphic

6. If we label the diagonals in the square by 1 and 2, every isometry of the square would permute them. This gives a homomorphism from  $D_4$  to  $S_2$ . Describe its kernel and image.