## MAT 150A, Fall 2017 <br> Practice problems for Midterm 2

1. Prove that every group with 4 elements is isomorphic either to $\mathbb{Z}_{4}$ or to $\mathbb{Z}_{2} \times \mathbb{Z}_{2}$ (and is therefore commutative).
2. Are the following matrices orthogonal? Do they preserve orientation? Describe the corresponding transformations geometrically.

$$
\left(\begin{array}{ll}
1 & 1 \\
1 & 1
\end{array}\right),\left(\begin{array}{ll}
0 & 1 \\
1 & 0
\end{array}\right),\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right),\left(\begin{array}{cc}
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{array}\right),\left(\begin{array}{llll}
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0
\end{array}\right)
$$

3. Let $A$ be the matrix of reflection in $x-y$ plane, and let $B$ be the matrix of reflection in $y-z$ plane.
a) Write the $3 \times 3$ matrices $A$ and $B$ explicitly.
b) Compute the matrix products $A B$ and $B A$.
c) Prove that $A B$ and $B A$ are rotations and find the axis of these rotations.
4. Describe all homomorphisms (a) From $\mathbb{Z}_{5}$ to $\mathbb{Z}_{7}$ (b) From $\mathbb{Z}_{4}$ to $\mathbb{Z}_{6}$
5. Prove that the groups $\mathbb{Z}_{6}$ and $S_{3}$ are not isomorphic
6. If we label the diagonals in the square by 1 and 2 , every isometry of the square would permute them. This gives a homomorphism from $D_{4}$ to $S_{2}$. Describe its kernel and image.
