

MAT 150A, Fall 2021
Practice problems for Midterm 1

1. Decompose the product of permutations into non-intersecting cycles:

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 6 & 1 & 3 & 2 & 5 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 2 & 3 & 4 & 6 & 1 \end{pmatrix}$$

2. Consider the permutation:

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 7 & 5 & 2 & 8 & 3 & 4 & 1 & 10 & 6 & 9 \end{pmatrix}$$

- (a) Decompose f into non-intersecting cycles;
(b) Compute the sign of f ;
(c) Compute the order of f ;
(d) Compute the inverse permutation f^{-1} .
3. Find permutations in S_7 of orders 10, 11, 12 or prove that there are none.
4. Is the set of even integers a subgroup of $(\mathbb{Z}, +)$? The set of odd integers?
5. Find the orders of all elements in \mathbb{Z}_6 .
6. Solve the equation $8x = 1 \pmod{11}$.
7. Find the orders of all elements in \mathbb{Z}_{11}^* and prove that this group is cyclic.
8. Find all finite subgroups of \mathbb{R}^* . *Hint: find all elements of finite order.*
9. Give an example of a group G and two elements x, y in G such that

$$\text{Ord}(xy) \neq \text{LCM}(\text{Ord}(x), \text{Ord}(y)).$$

- 10**. Give an example of a group G and two elements x, y in G such that x and y have finite orders, but xy has infinite order.