

MAT 150A, Fall 2021  
Homework 2

**Due before 12:10 on Wednesday, October 6**

*Please write the homework solutions in connected sentences and explain your work. Mark the answers to each question. Scan or take pictures of your homework and upload it to Gradescope before due time.*

1. (10 points) Let  $x, y, z$  and  $w$  be elements of a group  $G$ .
  - a) Solve for  $y$  given that  $xyz^{-1}w = 1$ .
  - b) Suppose that  $xyz = 1$ . Does it follow that  $yzx = 1$ ? Does it follow that  $yxz = 1$ ?
2. How many elements of order 2 does the symmetric group  $S_4$  contain?
3. Find the order of the permutation

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 3 & 10 & 7 & 6 & 2 & 5 & 9 & 4 & 1 & 8 \end{pmatrix}$$

4. Determine if the following subset  $H$  is a subgroup of  $G$ :
  - a)  $G = (\mathbb{R}^*, \cdot)$  and  $H = \{-1, 1\}$ .
  - b)  $G = (\mathbb{Z}, +)$  and  $H$  is the set of positive integers.
  - c)  $G = (\mathbb{R}^*, \cdot)$  and  $H$  is the set of positive real numbers.