

MAT 150A, Fall 2023  
Homework 3

**Due before 12:10 on Wednesday, October 18**

*Please write the homework solutions in connected sentences and explain your work. Mark the answers to each question. Scan or take pictures of your homework and upload it to Gradescope before due time.*

1. Let  $x$  and  $y$  be elements of a group  $G$ . Assume that each of the elements  $x, y$  and  $xy$  has order 2.
  - a) Prove that  $xy = yx$ .
  - b) Prove that the set  $H = \{1, x, y, xy\}$  is a subgroup of  $G$ .
2. Prove that every integer is equal to the sum of its decimal digits modulo 9.
3. Solve the equation  $2x = 5$  modulo 9 and modulo 6.
4. Compute  $2^{2021} \pmod{7}$ .