MAT 150A, Fall 2023
Homework 4

Due before 12:10 on Wednesday,November 1

Please write the homework solutions in connected sentences and explain your work. Mark the answers to each question. Scan or take pictures of your homework and upload it to Gradescope before due time.

1. Let G be a group of order 25. Prove that G has at least one subgroup of order 5, and that if it contains only one subgroup of order 5 then it is a cyclic group.

2. Is it possible to construct an injective homomorphism (a) from $\mathbb{Z}_3$ to $\mathbb{Z}_4$? (b) From S_3 to S_4?

3. A finite group G contains an element x of order 10 and also an element y of order 6. What can be said about the order of G?

4. Let $\varphi : G_1 \to G_2$ be a group homomorphism. Suppose that $|G_1| = 18, |G_2| = 15$ and that φ is not the trivial homomorphism. What is the order of the kernel?