MAT 150C, Spring 2017
Practice problems for Midterm 1

This practice sheet contains more problems than the actual exam.

1. Suppose that $\rho : G \to GL(V)$ is a representation of G and $f : H \to G$ is a group homomorphism. Define $\rho' : H \to GL(V),\ \rho'(h) = \rho(f(h))$.

(a) Prove that ρ' is a representation of H.

(b) Prove that if ρ' is irreducible then ρ is irreducible.

(c) Give an example where ρ is irreducible but ρ' is not.

2. Describe all irreducible representations for (a) $\mathbb{Z}_4$ (b) $\mathbb{Z}_2 \times \mathbb{Z}_2$.

3. Consider the permutation representation $S_3 \to GL(V), V = \mathbb{C}^3$.

(a) Compute the dimension of the space of S_3-invariant transformations from V to V.

(b) Find an explicit basis in this space.

4. Describe all conjugacy classes in the dihedral group D_n.

5. The action of D_n on the diagonals of the regular n-gon defines a representation of D_n. Compute the character of this representation and decompose it into irreducibles for (a) $n = 4$ (b) $n = 5$.

6*. Let G be a noncommutative group. Prove that it has an irreducible complex representation of dimension at least 2. *Hint: prove that the number of conjugacy classes in G is strictly less than $|G|$.*