1. Suppose that $\rho : G \to GL(V)$ is a representation of $G$ and $f : H \to G$ is a group homomorphism. Define $\rho' : H \to GL(V), \: \rho'(h) = \rho(f(h))$.

(a) Prove that $\rho'$ is a representation of $H$.

(b) Prove that if $\rho'$ is irreducible then $\rho$ is irreducible.

(c) Give an example where $\rho$ is irreducible but $\rho'$ is not.

2. Describe all irreducible representations for (a) $\mathbb{Z}_4$ (b) $\mathbb{Z}_2 \times \mathbb{Z}_2$.

3. Consider the permutation representation $S_3 \to GL(V), V = \mathbb{C}^3$.

(a) Compute the dimension of the space of $S_3$-invariant transformations from $V$ to $V$.

(b) Find an explicit basis in this space.

4. Describe all conjugacy classes in the dihedral group $D_n$.

5. The action of $D_n$ on the diagonals of the regular $n$-gon defines a representation of $D_n$. Compute the character of this representation and decompose it into irreducibles for (a) $n = 4$ (b) $n = 5$.

6*. Let $G$ be a noncommutative group. Prove that it has an irreducible complex representation of dimension at least 2. **Hint:** prove that the number of conjugacy classes in $G$ is strictly less than $|G|$. 