MAT 150C, Spring 2017
Homework Assignment 7

Due before the start of the class on Wednesday June 7

Please read Section 16 of the textbook before starting on the problem set.

Written Assignment: 1. Let F be a subfield in a field K. Define $G(K/F)$ to be the set of all automorphisms $\phi : K \to K$ such that $\phi(x) = x$ for x in F. Prove that $G(K/F)$ is a group.

2. Prove that the dihedral group D_n is solvable.

3. Let $f(x)$ be an irreducible polynomial with rational coefficients of degree 3.

a) Prove that the degree of the splitting field K for $f(x)$ is equal to 3 or to 6.

b) If $[K : \mathbb{Q}] = 6$, prove that the Galois group of $f(x)$ is isomorphic to S_3

c) If $[K : \mathbb{Q}] = 3$, prove that the Galois group of $f(x)$ is isomorphic to $\mathbb{Z}_3$

d) Prove that in both cases the Galois group is solvable, and conclude that every equation of degree 3 can be solved in radicals.

The homework must be legible, and written in connected sentences that explains what you are doing. Just the answer (whether correct or not) is not enough. Please put your name and section number on every page and staple the pages together. Homework should be handed in on time, late homework will not be graded.