1. Find the minimal polynomial for $\sqrt{2} + \sqrt{3}$ over $\mathbb{Q}$.
2. Find all irreducible polynomials over $\mathbb{Z}_2$ of degrees (a) 1 (b) 2 (c) 3 (d) 4.
3. Prove that the polynomial $x^3 - 3x + 1$ is irreducible over $\mathbb{Q}$.
4. Decompose the polynomial $x^4 + 1$ into irreducible factors over (a) $\mathbb{Q}$ (b) $\mathbb{Q}[\sqrt{2}]$ (c) $\mathbb{Q}[i]$.
5. Compute $\frac{1}{1 + \sqrt{2}}$ in the field $\mathbb{Q}[\sqrt{2}]$.
6. Let $\chi_n : SU(2) \to \mathbb{C}$ be the character of the $(n+1)$ dimensional irreducible representation $V_n$.
   (a) Prove that $f(A) = \chi_1(A) \cdot \chi_3(A)$ can be written as a sum of $\chi_i$ with nonnegative integer coefficients.
   (b) Construct a representation of $SU(2)$ with character $f(A)$.
7*. Suppose that $p > 2$ is a prime number, consider the field $F = \mathbb{Z}_p$.
   (a) Consider the function $f : F \to F$, $f(x) = x^2$. Prove that every nonzero element has either 0 or 2 preimages.
   (b) Use (a) to compute the number of complete squares in $F$
   (c) Use (b) to prove that for some $a \in F$ the polynomial $x^2 - a$ is irreducible.
   (d) Prove that there exists a field with $p^2$ elements.