MAT 17BA Fall 2023
Solutions to homework 2

1. (10 points) The formula $C = \frac{5}{9}(F - 32)$ expresses the Celsius temperature C as a function of the Fahrenheit temperature. Find a formula for the inverse function and interpret it.

Solution: We solve for F and get

$$F - 32 = \frac{9}{5}C, \ F = \frac{9}{5}C + 32.$$

The inverse function expresses the Fahrenheit temperature as a function of the Celsius temperature.

2. (10 points) The table gives the midyear population of India (in millions):

Year	1950	1960	1970	1980	1990	2000
Population	370	445	554	685	838	1006

a) Make a scatter plot, semilog plot, and log-log plot for these data.
b) Find a linear function which approximates the semilog plot.
c) Use the result of part (b) to find an exponential model for the population.

Solution: First, we compute the logarithms for both axis:

Year	1950	1960	1970	1980	1990	2000
Log(Year)	3.290	3.292	3.294	3.297	3.299	3.301
Population	370	445	554	685	838	1006
Log(Population)	2.568	2.648	2.744	2.836	2.923	3.003

The plots are shown on the next page. To find a linear function approximating the semilog plot, we can find the equation of a line connecting the first and last point (dashed) which seems to approximate the data points well. Its slope equals

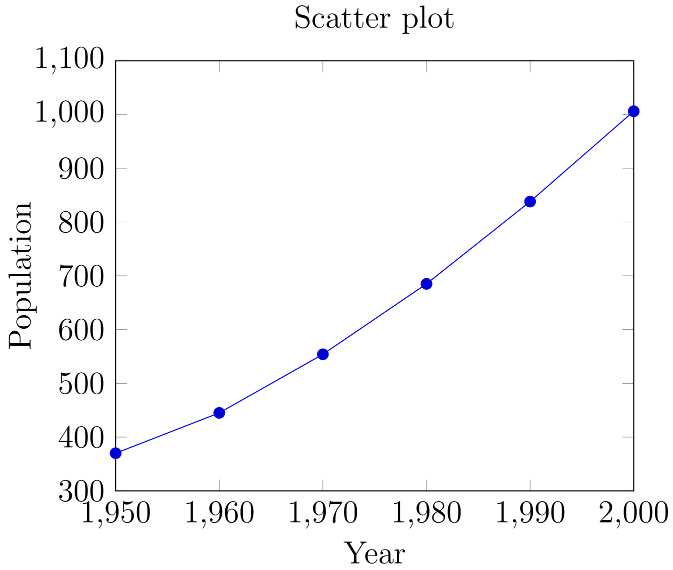
$$k = \frac{3.003 - 2.568}{2000 - 1950} \approx 0.0087,$$

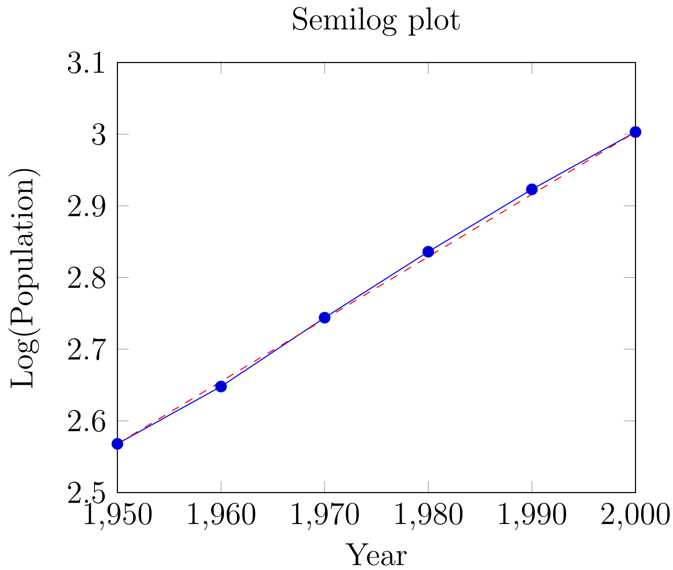
so the equation in semilog plot is

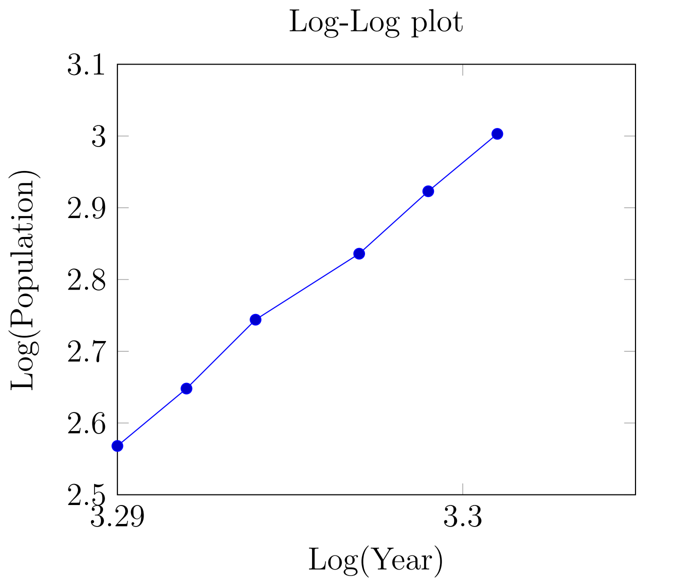
$$\text{Log}(y) \approx 0.0087(x - 1950) + 2.568 = 0.0087x - 14.397$$

By exponentiating, we get the model

$$y(x) \approx 10^{0.0087(x-1950)+2.568} = 10^{0.0087x-14.397}.$$


Scatter plot
1,100
1,000
900
800
700
600
500
400
300
Population
1,950
1,960
1,970
1,980
1,990
2,000
Year


Semilog plot
3.1
3
2.9
2.8
2.7
2.6
2.5
Log(Population)
1,950
1,960
1,970
1,980
1,990
2,000
Year

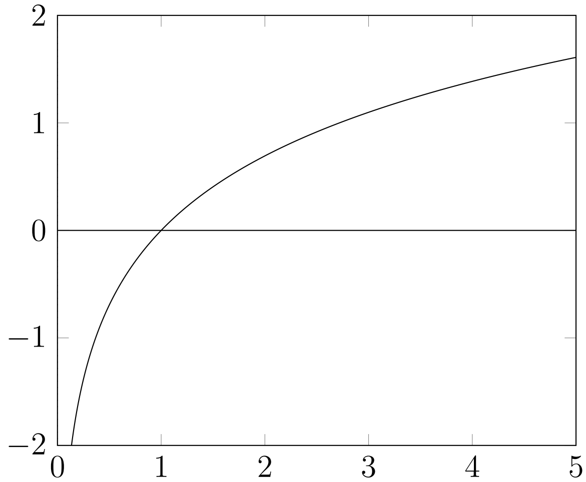

Log-Log plot
3.1
3
2.9
2.8
2.7
2.6
2.5
Log(Population)
3.29
3.3
Log(Year)

3. (10 points) Sketch the graph of the function $\ln(100x^5)$ using transformations. *Hint: you might want to simplify the function first*

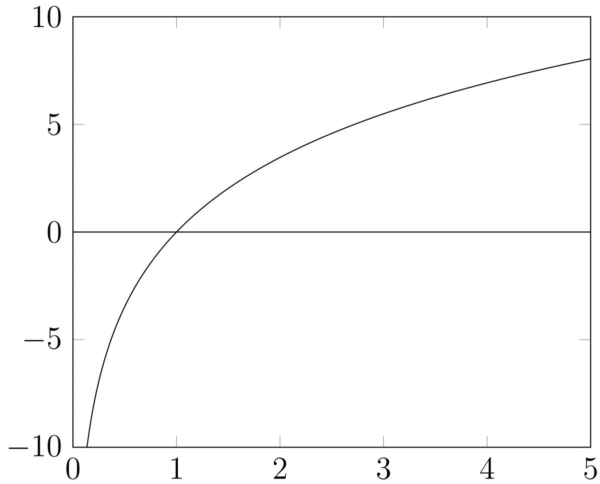
Solution: First, we can simplify the function:

$$\ln(100x^5) = \ln(100) + \ln(x^5) = \ln(100) + 5\ln(x).$$

Now we can sketch the graph starting from the standard graph of $\ln(x)$:



Next we stretch it vertically by a factor of 5 to get the graph of $5\ln(x)$:



Next we shift it up by $\ln(100)$ to get the graph of $5\ln(x) + \ln(100)$: