MAT 17A Fall 2023
Solutions to homework 3

1. (10 points) Compute the limit $\lim_{x\to\infty} \frac{x^2-4x}{x^2-3x-4}$.

Solution: We divide the numerator and the denominator of the fraction by x^2 which is the highest power of x:

$$\lim_{x\to\infty} \frac{x^2-4x}{x^2-3x-4} = \lim_{x\to\infty} \frac{1-4/x}{1-3/x-4/x^2} = \frac{1-0}{1-0-0} = 1.$$

Here we used that $\lim_{x\to\infty} 1/x = \lim_{x\to\infty} 1/x^2 = 0$.

2. (10 points) Compute the limit $\lim_{x\to 0} \frac{x^2-4x}{x^2-3x-4}$.

Solution: The function is continuous at $x = 0$, so we can simply write

$$\lim_{x\to 0} \frac{x^2-4x}{x^2-3x-4} = \frac{0^2-4\cdot 0}{0^2-3\cdot 0-4} = \frac{0}{-4} = 0.$$

3. (10 points) For the function $f(x) = \frac{x^2+2}{x^2-1}$:

a) Find the domain.

b) Find the vertical asymptotes.

c) Find the horizontal asymptotes.

Solution: a) The function is defined when $x^2 - 1 \neq 0$, so $x^2 \neq 1$ and $x \neq \pm 1$. Therefore the domain is $(-\infty, -1) \cup (-1, 1) \cup (1, +\infty)$.

b) We have

$$\lim_{x\to 1}(x^2+2) = 3,\ \lim_{x\to 1}(x^2-1) = 0,\ \text{so}\ \lim_{x\to 1} \frac{x^2+2}{x^2-1} = \infty.$$

Similarly,

$$\lim_{x\to -1}(x^2+2) = 3,\ \lim_{x\to -1}(x^2-1) = 0,\ \text{so}\ \lim_{x\to -1} \frac{x^2+2}{x^2-1} = \infty.$$

Therefore $f(x)$ has vertical asymptotes $x = 1$ and $x = -1$.

c) We have

$$\lim_{x\to\infty} \frac{x^2+2}{x^2-1} = \lim_{x\to\infty} \frac{1+2/x^2}{1-1/x^2} = \frac{1+0}{1-0} = 1,$$

and similarly $\lim_{x\to-\infty} \frac{x^2+2}{x^2-1} = 1$. Therefore $f(x)$ has a horizontal asymptote $y = 1$ at $+\infty$ and at $-\infty$.