

MAT 17A Fall 2023
Solutions to Homework 4

Find the derivatives of the following functions:

1. (10 points)

$$f(x) = \frac{x^2 + 4x + 3}{\sqrt{x}}$$

Solution 1: We can write $f(x) = \frac{x^2+4x+3}{\sqrt{x}} = x^{3/2} + 4x^{1/2} + 3x^{-1/2}$, so by Power Rule

$$f'(x) = \frac{3}{2}x^{1/2} + 2x^{-1/2} - \frac{3}{2}x^{-3/4}.$$

Solution 2: We apply Quotient Rule:

$$\begin{aligned} f'(x) &= \frac{(x^2 + 4x + 3)' \sqrt{x} - (x^2 + 4x + 3)(\sqrt{x})'}{(\sqrt{x})^2} = \\ &= \frac{(2x + 4)\sqrt{x} - (x^2 + 4x + 3)\frac{1}{2\sqrt{x}}}{(\sqrt{x})^2} = \frac{(2x + 4)2x - (x^2 + 4x + 3)}{2x\sqrt{x}} = \\ &= \frac{4x^2 + 8x - x^2 - 4x - 3}{2x\sqrt{x}} = \frac{3x^2 + 4x - 3}{2x\sqrt{x}}. \end{aligned}$$

2. (10 points)

$$f(x) = \frac{1 - xe^x}{x + e^x}$$

Solution: We apply Quotient Rule:

$$\begin{aligned} f'(x) &= \frac{(1 - xe^x)'(x + e^x) - (1 - xe^x)(x + e^x)'}{(x + e^x)^2} = \\ &= \frac{(-e^x - xe^x)(x + e^x) - (1 - xe^x)(1 + e^x)}{(x + e^x)^2} = \\ &= \frac{-xe^x - e^{2x} - x^2e^x - xe^{2x} - 1 - e^x + xe^x + xe^{2x}}{(x + e^x)^2} = \\ &= \frac{-e^{2x} - x^2e^x - 1 - e^x}{(x + e^x)^2}. \end{aligned}$$

3. (10 points)

$$f(x) = \sin(x^2)$$

Solution: We apply Chain Rule:

$$f'(x) = \cos(x^2) \cdot 2x.$$