

MAT 17B, Fall 2020
Practice problems for final exam

This practice sheet contains more problems than the actual exam

1. Compute the following integrals:

a) $\int_1^2 \sqrt{x} dx$

b) $\int \frac{dx}{3x-5}$

c) $\int \frac{x dx}{\sqrt{4-x^2}}$

d) $\int (3x-5)\sqrt{x} dx$

e) $\int x \sin x dx$

f) $\int_2^3 \frac{dx}{x \ln^2 x}$

2. Find the area of the region bounded by the curves $y = x^2 - x - 1$ and $y = 4x - 7$.

3. Find the volume of the solid of revolution obtained by rotation of the graph of $f(x) = e^x$ on the interval $[2, 5]$ around the x -axis.

4. Solve the differential equations:

a) $y' = y(x-1)$

b) $y' = 3y - 2$

c) $y' = \frac{\sin x}{\sin y}$

d) $y' = \frac{1+y^2}{xy}$

5. Consider the differential equation $y' = -(y+1)(y-2)$.

a) Find all equilibrium solutions

b) Sketch the phase plot

c) Determine where the function $y(x)$ is increasing or decreasing

d) Determine if the equilibria are stable or unstable.

6. Consider the vectors $u = (3, 0, -1)$ and $v = (7, 5, 2)$.

a) Find the vector $3u + v$

b) Find the dot product $u \cdot v$

c) Find the lengths of u and v

d) Find the angle between u and v

7. Consider the matrices

$$A = \begin{pmatrix} 4 & -2 \\ 3 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix}.$$

a) Find matrices $A + B$ and $3A - 2B$

b) Find matrices A^2, B^2, AB and BA

c) Are matrices A and B invertible? If they are, find their inverses.

8. Find the eigenvectors and eigenvalues for the following matrices:

(a) $\begin{pmatrix} -2 & 1 \\ 0 & 3 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 4 & -2 \\ 3 & -1 \end{pmatrix}$

9*. Consider the matrix $C = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. Compute C^{100} .