

MAT 17B, Fall 2020

Homework 5

Due before 4:10 PM on Friday, November 13

Please write the homework solutions in connected sentences and explain your work. Mark the answers to each question. Scan or take pictures of your homework and upload it to Gradescope before due time.

1. (10 points) Sketch the phase plot for the autonomous differential equation $y' = y(3 - y)(25 - y^2)$. Find all equilibria and determine if each is locally stable or unstable.
2. (10 points) Sketch the phase plot for the autonomous differential equation $y' = 5y(2e^{-y} - 1)$. Find all equilibria and determine if each is locally stable or unstable.
3. (10 points) The population size $N(t)$ satisfies the differential equation

$$N' = 2\left(1 - \frac{N}{1000}\right)N - hN,$$

where h is a parameter called *harvest rate*. Assume $h \neq 2$.

- a) Find all equilibria for this equation (the answer depends on h).
- b) Determine if these equilibria are stable or unstable (the answer depends on h).
- c) Find the limit $\lim_{t \rightarrow \infty} N(t)$ depending on the initial condition $N(0) > 0$ (the answer depends on h).