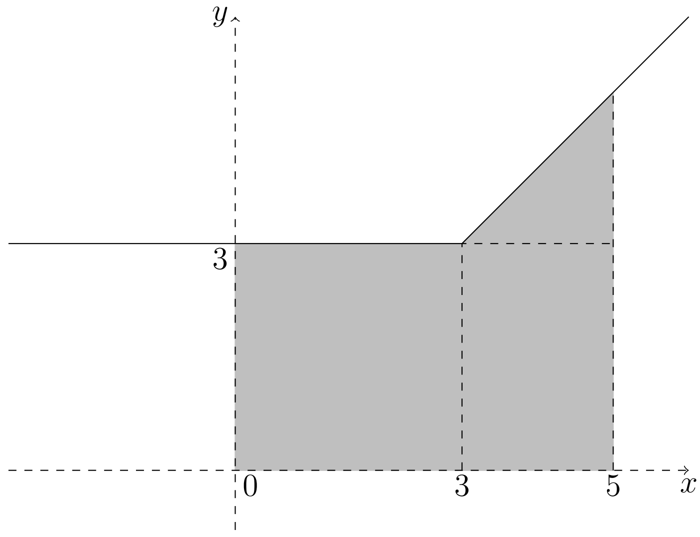
MAT 17B, Spring 2020

Solutions to Homework 1

1. (10 points) (a) (5 points) Draw the graph of

$$f(x) = \begin{cases} 3 & \text{for } x < 3 \\ x & \text{for } x \geq 3 \end{cases}$$

Solution: The function is constant for $x < 3$ and linear for $x \geq 3$. The limits from the left and right agree at $x = 3$, and both are equal to 3, so $f(x)$ is continuous everywhere.



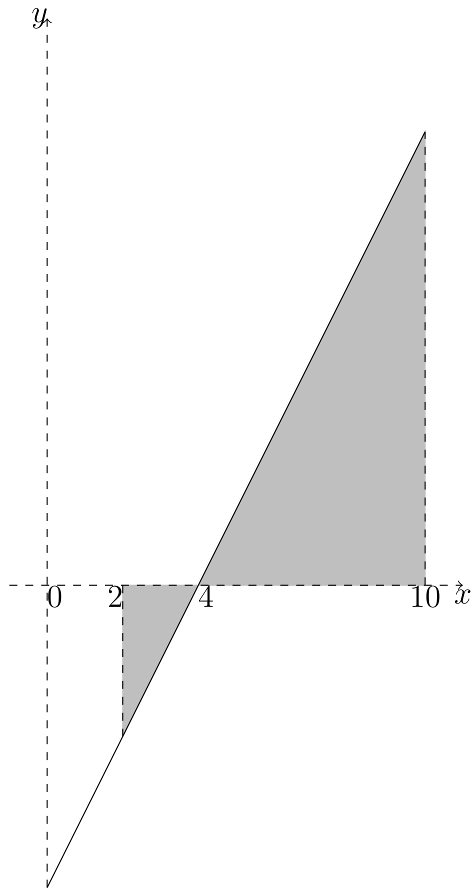
(b) (5 points) Compute the integral $\int_0^5 f(x)dx$ by interpreting it in terms of areas.

Solution: The area below the graph of $f(x)$ on the interval $[0, 5]$ can be decomposed into a 3×3 square, a 2×3 rectangle, and a triangle with base and height 2. Therefore the total area equals

$$\int_0^5 f(x)dx = 3 \cdot 3 + 2 \cdot 3 + \frac{1}{2} \cdot 2 \cdot 2 = 9 + 6 + 2 = 17.$$

2. (10 points) Compute the integral $\int_2^{10} (2x - 8)dx$ by interpreting it in terms of areas.

Solution: The graph of the function $f(x) = 2x - 8$ has the following form:



It intersects the x-axis at $x = 4$. The area between the graph and the x-axis on the interval $[2, 10]$ consists of two triangles:

- The first has base $4 - 2 = 2$ and height $|f(2)| = 4$, so it has area $\frac{1}{2} \cdot 2 \cdot 4 = 4$
- The second has base $10 - 4 = 6$ and height $|f(10)| = 12$, so it has area $\frac{1}{2} \cdot 6 \cdot 12 = 36$.

Therefore

$$\int_2^{10} (2x - 8)dx = 36 - 4 = 32.$$

3. (10 points) (a) (5 points) Estimate the integral $\int_{-2}^{2} x^2 dx$ using four intervals and midpoints.

Solution: The interval $[-2, 2]$ can be divided into four equal intervals $[-2, -1], [-1, 0], [0, 1]$ and $[1, 2]$. The corresponding midpoints are at $-1.5, -0.5, 0.5$ and 1.5, and the integral sum equals

$$M_4 = (f(-1.5)+f(-0.5)+f(0.5)+f(1.5))\cdot 1 = 2.25+0.25+0.25+2.25 = 5.$$

(b)(5 points) Use Fundamental Theorem of Calculus to compute $\int_{-2}^{2} x^2 dx$ exactly.

Solution: We have

$$\int_{-2}^{2} x^2 dx = \frac{x^3}{3}|_{-2}^{2} = \frac{8}{3} - \frac{(-8)}{3} = \frac{16}{3} = 5\frac{1}{3}.$$