MAT 17B, Fall 2020
Solutions to Homework 2

1. (10 points) Compute the integral $\int_0^{18} \sqrt{\frac{3}{z}} dz$.

Solution: We have

$$\int_0^{18} \sqrt{\frac{3}{z}} dz = \int_0^{18} \sqrt{3} \cdot z^{-\frac{1}{2}} dz = \sqrt{3} \cdot \frac{z^{1/2}}{1/2} |_0^{18} =$$

$$2\sqrt{3}\sqrt{z}|_0^{18} = 2\sqrt{3}\sqrt{18} - 0 = 2\sqrt{54} = 6\sqrt{6}.$$

2. (10 points) Many fish grow in a way that is described by the von Bertalanffy growth equation. For a fish that starts life with a length of 1 cm and has a maximum length of 30 cm, this equation predicts that the growth rate is $29e^{-a}$ cm/year, where a is the age of the fish. How long will the fish be after 5 years?

Solution 1: Let $L(t)$ be the length of fish in cm after t years, then $L(0) = 1$ and $L'(t) = 29e^{-t}$. By Net Change formula (or Fundamental Theorem of Calculus) we get

$$L(5) = L(0) + \int_0^5 L'(t)dt = 1 + \int_0^5 29e^{-t}dt = 1 - 29e^{-t}|_0^5 =$$

$$1 - (29e^{-5} - 29e^0) = 1 - 29e^{-5} + 29 = 30 - 29e^{-5}.$$

Solution 2: As above, we have $L'(t) = 29e^{-t}$, so $L(t)$ is an antiderivative of $29e^{-t}$ and hence

$$L(t) = -29e^{-t} + C$$

where C is some constant. We can determine this constant from the initial condition:

$$L(0) = -29e^0 + C = C - 29 = 1,$$

so $C = 30$. Therefore $L(t) = 30 - 29e^{-t}$ and $L(5) = 30 - 29e^{-5}$.

3. (10 points) Compute the integral $\int x^2(x^3+5)^9 dx$ using u-substitution.

Solution: Let $u = x^3 + 5$, then $du = 3x^2 dx$ and $x^2 dx = \frac{1}{3} du$. Now

$$\int x^2(x^3+5)^9 dx = \int u^9 \cdot \frac{1}{3} du = \frac{1}{3} \int u^9 du = \frac{1}{3} \cdot \frac{u^{10}}{10} + C =$$

$$\frac{u^{10}}{30} + C = \frac{(x^3+5)^{10}}{30} + C.$$