MAT 17B, Fall 2020
Solutions to homework 3

1. (10 points) Compute the integral $\int_0^{\pi/2} \cos(x) \cdot \sin(\sin(x))dx$.

Solution: Let $u = \sin(x)$, then $du = \cos(x)dx$ and

$$\int_0^{\pi/2} \cos(x) \cdot \sin(\sin(x))dx = \int_0^1 \sin(u)du =$$

$$(-\cos(u))|_0^1 = -\cos(1) - (-\cos(0)) = -\cos(1) + 1.$$

Note that if $x = 0$ then $u = \sin(0) = 0$ and if $x = \pi/2$ then $u = \sin(\pi/2) = 1$.

2. (10 points) Compute the integral $\int_0^1 \frac{e^z+1}{e^z+z}dz$.

Solution: Let $u = e^z + z$, then $du = (e^z + 1)dz$ and

$$\int_0^1 \frac{e^z + 1}{e^z + z}dz = \int_1^{e+1} \frac{1}{u}du = (\ln|u|)_1^{e+1} =$$

$$\ln(e + 1) - \ln(1) = \ln(e + 1).$$

Note that if $x = 0$ then $u = e^0 + 0 = 1$ and if $x = 1$ then $u = e^1 + 1 = e + 1$.

3. (10 points) Compute the integral $\int_0^{\pi} t\sin(3t)dt$.

Solution: We use integration by parts. Let $u = t$ and $dv = \sin(3t)dt$, then $v = -\frac{1}{3}\cos(3t)$ and $du = dt$. Now

$$\int t\sin(3t)dt = \int udv = uv - \int vdu =$$

$$t \cdot (-\frac{1}{3}\cos(3t)) - \int(-\frac{1}{3}\cos(3t))dt = -\frac{1}{3}t\cos(3t) + \frac{1}{3}\int\cos(3t)dt =$$

$$-\frac{1}{3}t\cos(3t) + \frac{1}{3} \cdot \frac{1}{3}\sin(3t) = -\frac{1}{3}t\cos(3t) + \frac{1}{9}\sin(3t).$$

Now

$$\int_0^{\pi} t\sin(3t)dt = \left[-\frac{1}{3}t\cos(3t) + \frac{1}{9}\sin(3t)\right]_0^{\pi} =$$

$$\left[-\frac{1}{3}\pi\cos(3\pi) + \frac{1}{9}\sin(3\pi)\right] - \left[-\frac{1}{3}0\cos(0) + \frac{1}{9}\sin(0)\right] =$$

$$-\frac{1}{3}\pi(-1) + 0 - 0 - 0 = \frac{1}{3}\pi.$$