MAT 17B, Fall 2020
Solutiond to homework 6

1. (10 points) $y' = xe^{-y}$

Solution: We have $\frac{dy}{dx} = xe^{-y}$, so

$$\int e^y dy = \in x dx, \; e^y = \frac{x^2}{2} + C,$$

so

$$y = \ln\left(\frac{x^2}{2} + C\right).$$

2. (10 points) $y' = y^2$.

Solution: We have $\frac{dy}{dx} = y^2$, so

$$\int y^{-2} dy = \int dx, \; -y^{-1} = x + C,$$

so

$$y = \frac{-1}{x + C}.$$

3. (10 points) $y' = \frac{\ln x}{xy}, \; y(1) = 2$

Solution: We have $\frac{dy}{dx} = \frac{\ln x}{xy}$, so

$$\int y dy = \int \frac{\ln x}{x} dx.$$

For the right hand side, let us make a u-substitution $u = \ln x$, then $du = \frac{1}{x} dx$, so

$$\int \frac{\ln x}{x} dx = \int u du = \frac{u^2}{2} + C = \frac{\ln(x)^2}{2} + C.$$

Therefore

$$\frac{y^2}{2} = \frac{\ln(x)^2}{2} + C, \; y^2 = \ln(x)^2 + 2C,$$

and the general solution has the form

$$y(x) = \pm\sqrt{\ln(x)^2 + 2C}.$$

Since $y(1) = 2$ and $\ln(1) = 0$, we get $2^2 = 4 = 0 + 2C$, so $C = 2$ and

$$y(x) = \pm\sqrt{\ln(x)^2 + 4}.$$