

MAT 17B, Fall 2020
Solutions to homework 7

1. (10 points) Consider the vectors $u = (1, -2, 3)$ and $v = (7, -2, 1)$. Find the lengths of vectors u , v and $u + v$.

Solution: We have $u + v = (1 + 7, -2 - 2, 3 + 1) = (8, -4, 4)$, so

$$|u| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{14}, \quad |v| = \sqrt{7^2 + (-2)^2 + 1^2} = \sqrt{54}$$

and

$$|u + v| = \sqrt{8^2 + (-4)^2 + (-4)^2} = \sqrt{96}.$$

2. (10 points) Find the distance between the points $A = (1, -1)$ and $B = (7, 7)$.

Solution: The vector \overline{AB} has coordinates $(7 - 1, 7 - (-1)) = (6, 8)$, so its length equals

$$|\overline{AB}| = \sqrt{6^2 + 8^2} = \sqrt{100} = 10.$$

3. (10 points) Find the angle between the vectors $(1, 1, 0)$ and $(1, 0, 1)$.

Solution: Both vectors have length $\sqrt{1^2 + 1^2 + 0^2} = \sqrt{2}$, and their dot product equals $1 \cdot 1 + 1 \cdot 0 + 0 \cdot 1 = 1$. Therefore we have

$$\cos(\alpha) = \frac{u \cdot v}{|u||v|} = \frac{1}{\sqrt{2}\sqrt{2}} = \frac{1}{2},$$

and $\alpha = \arccos(\frac{1}{2}) = 60^\circ$.