MAT 17C, Fall 2017

Practice problems for Midterm 2

This practice sheet has more problems than the actual exam

1. Find all critical points for a function of two variables. Determine the type (local minimum, local maximum, saddle) for each of them.

a) $f(x,y) = xy$

b) $f(x,y) = xe^y$

c) $f(x,y) = x^2 + 2xy + 3y^2 + 3y + 5$

d) $f(x,y) = x^3 - 3x + y^3 - 3y$.

2. Find the eigenvalues and the eigenvectors of the following matrices:

a) $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

b) $A = \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix}$

c) $A = \begin{pmatrix} 1 & 3 \\ 1 & -1 \end{pmatrix}$

3. Solve the linear systems of differential equations:

a) $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

b) $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

c) $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

d) $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 9 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

4. Solve the initial value problems:

a) $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$, $x(0) = 1, y(0) = 2$.

b) $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$, $x(0) = -1, y(0) = 2$.

5*. A 2×2 matrix of the form $A = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$ is called *symmetric*. Show that a symmetric matrix always has two real eigenvalues.

(continued on the next page)

6. Sketch the phase portraits for the following linear systems of differential equations. For each of them determine the type (sink, source, saddle, stable/unstable spiral, center), and determine if the system is stable.

a) $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ -2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

b) $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

c) $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 5 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

d) $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$