1. Prove that the limit
\[
\lim_{(x,y) \to (0,0)} \frac{x^2 - 5y^3}{x^2 + 3y^3}
\]
does not exist.

2. Compute the partial derivatives of the functions:
   a) \( x^2 - y^2 \)
   b) \( \frac{\sin(x+y)}{\cos(x-y)} \)
   c) \( \ln(x + e^y) \).

3. Find the gradient of the function \( \ln(x + e^y) \) at the point \((1, 0)\).

4. Use implicit differentiation to find the slope of the tangent line to the curve \( x^3 + y^3 = 28 \) at the point \((1, 3)\).

5. Find the directional derivative of the function \( f(x, y) = e^{x}e^y - xy \) at the point \((1, 2)\) in direction \( u = (1, 1) \).

6. Find all the critical points of a given function and determine their type (local minimum, local maximum, saddle):
   a) \( f(x, y) = x \ln(x) + y^2 \)
   b) \( f(x, y) = x^2 + xy + y^2 \)
   c) \( f(x, y) = e^{-x^2 - y^2} \).

7. Find the equation of the tangent plane to the graph of the function
\[
f(x, y) = x^3 + ye^x
\]
at the point \((1, 0)\).

8. Solve the system of linear differential equations (if the eigenvalues are real) and sketch the phase portrait:
   a) \[
   \begin{pmatrix}
   x'(t) \\
   y'(t)
   \end{pmatrix} = \begin{pmatrix}
   1 & 2 \\
   2 & 1
   \end{pmatrix} \begin{pmatrix}
   x(t) \\
   y(t)
   \end{pmatrix}
   \]
b) \[
\begin{pmatrix}
x'(t) \\
y'(t)
\end{pmatrix} = \begin{pmatrix} 0 & 5 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}
\]

9. Solve the second order linear differential equations:
   a) \[x''(t) + 3x'(t) + 2x(t) = 0\]
   b) \[x''(t) = x(t)\]
   c) \[x''(t) - x'(t) = 0, \quad x(0) = 1, \quad x'(0) = 2.\]

10. Solve the initial value problem:
    \[
    \begin{pmatrix} x'(t) \\ y'(t) \end{pmatrix} = \begin{pmatrix} 0 & 3 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}, \quad x(0) = 2, \quad y(0) = 0.
    \]

11. Let \(N(t)\) denote the population of rabbits and \(P(t)\) denote the population of wolves. This predator-prey system is described by a system of differential equations:
    \[
    \frac{dN}{dt} = 3N - PN, \quad \frac{dP}{dt} = -2P + PN.
    \]
    (a) Find all equilibrium points for this system; (b) Determine the type and stability of each equilibrium; (c) Sketch the phase portrait of the system.

12. Let \(N_1(t)\) and \(N_2(t)\) denote the populations of two species competing for limited resources. They are described by a system of differential equations:
    \[
    \begin{align*}
    \frac{dN_1}{dt} &= N_1(1 - \frac{N_1}{35} - 3\frac{N_2}{35}), \\
    \frac{dN_2}{dt} &= 3N_2(1 - \frac{N_2}{40} - 4\frac{N_1}{40}).
    \end{align*}
    \]
    (a) Find all equilibrium points for this system; (b) Determine the type and stability of each equilibrium; (c) Sketch the phase portrait of the system.

13. In a model for the forest growth, let \(x_1(t)\) and \(x_2(t)\) denote the areas occupied by the gaps and trees respectively. They are described by a compartment model:
    \[
    \begin{align*}
    \frac{dx_1}{dt} &= -0.2x_1 + 0.1x_2, \\
    \frac{dx_2}{dt} &= 0.2x_1 - 0.1x_2.
    \end{align*}
    \]
    Find a general solution for this system.

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14. Consider the system of differential equations:

\[ x'(t) = y(t), \quad y'(t) = -x(t). \]

a) Show that

\[ x(t) = C_1 \sin t + C_2 \cos t, \quad y(t) = C_1 \cos t - C_2 \sin t \]

is a solution of this system for arbitrary constants \( C_1, C_2. \)

b) Let \( F(t) = x(t)^2 + y(t)^2. \) Compute \( F'(t) \) in terms of \( x(t) \) and \( y(t). \)

c) Solve the system with the initial conditions \( x(0) = 3, y(0) = 5. \)