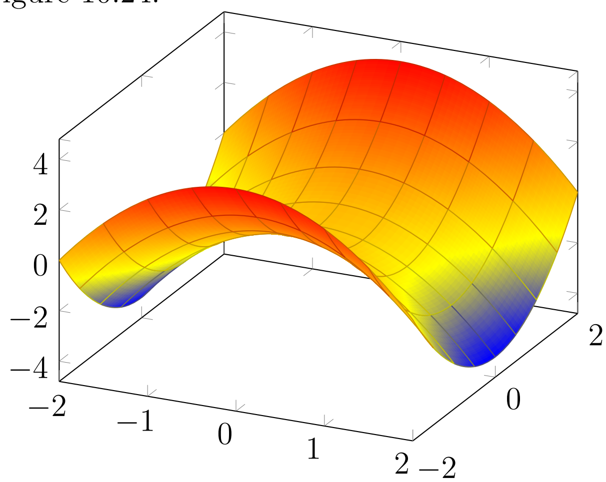
MAT 17C, Fall 2017
Solutions to Homework 1

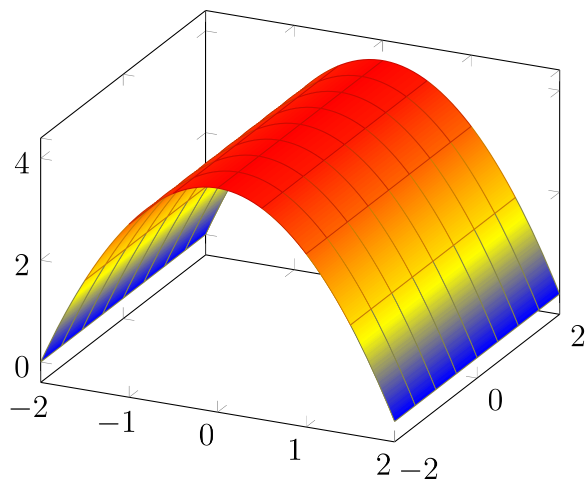
Chapter 10.1: 21. (20 points) Match the function $f(x, y) = y^2 - x^2$ to its graph.

Solution: At $y = 0$ we get $f(x, y) = -x^2$, this is a parabola extending downward. At $x = 0$ we get $f(x, y) = y^2$, this is a parabola extending upward. The only graph containing two such oppositely oriented parabolas is Figure 10.24.



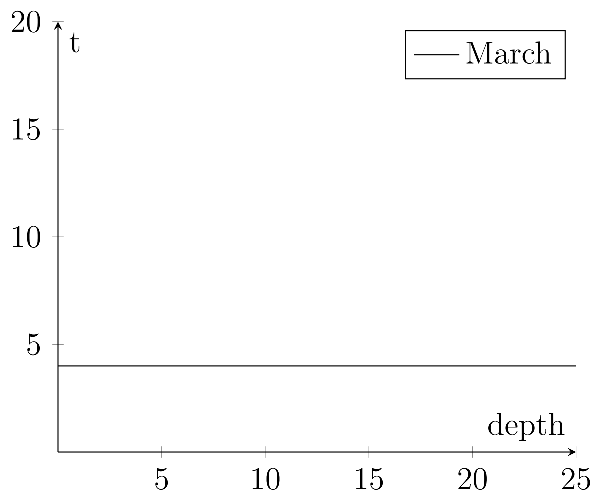
22.(20 points) Match the function $f(x, y) = 4 - x^2$ to its graph.

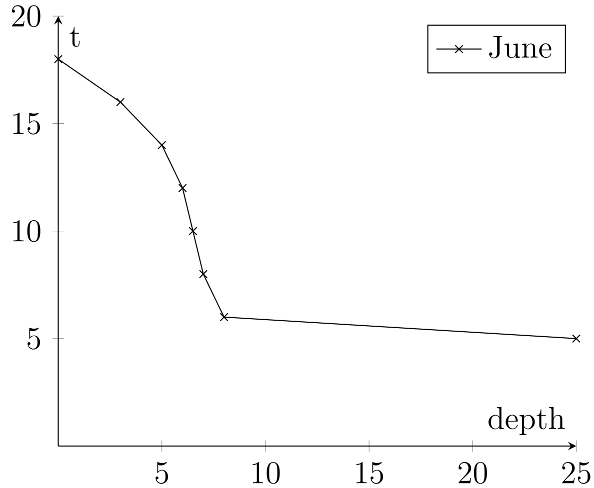
Solution: The function does not depend on y, so its value is unchanged as one travels in the y-direction. The only graph with this property is in Figure 10.21.



24.(20 points) Sketch the temperature profiles in March and January using Figure 10.25.

Solution:





Chapter 10.2: 16. (20 points) Show that the limit

$$\lim_{(x,y)\to(0,0)} \frac{3x^2 - y^2}{x^2 + y^2}$$

does not exist by computing the limit along the positive x-axis and the positive y-axis.

Solution: On the positive x-axis one has $y = 0$, so the limit equals

$$\lim_{x\to 0} \frac{3x^2 - 0^2}{x^2 + 0^2} = \lim_{x\to 0} 3 = 3.$$

On the positive y-axis one has $x = 0$, so the limit equals

$$\lim_{x\to 0} \frac{3 \cdot 0^2 - y^2}{0^2 + y^2} = \lim_{x\to 0}(-1) = -1.$$

Since the two limits along the axes are different, the limit does not exist.

Problem A: (20 points) Draw the contour plot for the function $f(x, y) = \sqrt{x^2 + y^2}$ by sketching the level curves $f(x, y) = c$ for $c = -3, -2, -1, 0, 1, 2, 3$.

Solution: Since $\sqrt{x^2 + y^2} \geq 0$ for all x and y, no points satisfy $f(x, y) = c$ for $c = -3, -2, -1$. For $c > 0$, we have

$$\sqrt{x^2 + y^2} = c \;\Leftrightarrow\; x^2 + y^2 = c^2.$$

This is the equation of a circle with center at the origin and radius c, so we get concentric circles of radius 0,1,2,3.