Section 10.3: Find the partial derivatives:

12. (20 points) $f(x, y) = e^{-y^2} \cos(x^2 - y^2)$

Solution: By the Product Rule and the Chain Rule we have

$$\frac{\partial f}{\partial x} = 0 \cdot \cos(x^2 - y^2) + e^{-y^2} \cdot (-\sin(x^2 - y^2)) \cdot (2x) = -2xe^{-y^2} \sin(x^2 - y^2),$$

$$\frac{\partial f}{\partial y} = e^{-y^2} \cdot (-2y) \cdot \cos(x^2 - y^2) + e^{-y^2} \cdot (-\sin(x^2 - y^2)) \cdot (-2y) =$$

$$= -2ye^{-y^2} \cos(x^2 - y^2) + 2ye^{-y^2} \sin(x^2 - y^2).$$

14. (20 points) $f(x, y) = \ln(3x^2 - xy)$.

Solution: By the Chain Rule we have

$$\frac{\partial f}{\partial x} = \frac{1}{3x^2 - xy} \cdot (6x - y), \quad \frac{\partial f}{\partial y} = \frac{1}{3x^2 - xy} \cdot (-x).$$

30. (20 points) The prey density $H(t)$ and the predator density $P(t)$ are related by the equation

$$\frac{1}{H} \frac{dH}{dt} = r \left(1 - \frac{H}{k}\right) - aP,$$

where $a, r, k$ are some positive constants. Investigate how an increase in (a) prey density and (b) predator density affects the per capita growth rate of this prey species.

Solution: The growth rate of prey species equals $\frac{dH}{dt}$, so per capita growth rate equals

$$\frac{1}{H} \frac{dH}{dt} = r \left(1 - \frac{H}{k}\right) - aP = r - rH/k - aP.$$

Let us denote the right hand side by $f(H, P)$, then

$$\frac{\partial f}{\partial H} = -r/k, \quad \frac{\partial f}{\partial H} = -a.$$

Therefore the increases in prey density or in predator density both decrease the per capita growth rate $f(H, P)$.
Section 10.4: 8. (20 points) Find the tangent plane to the graph of \( f(x, y) = e^x \cos y \) at the point \((0, 0, 1)\).

Solution: We have
\[
\frac{\partial f}{\partial x} = e^x \cos y, \quad \frac{\partial f}{\partial x}(0, 0) = 1 \cdot 1 = 1,
\]
\[
\frac{\partial f}{\partial y} = e^x (-\sin y), \quad \frac{\partial f}{\partial y}(0, 0) = 1 \cdot 0 = 0,
\]
so the equation of the tangent plane has the form
\[
z - z_0 = \frac{\partial f}{\partial x}(0, 0)(x - 0) + \frac{\partial f}{\partial y}(0, 0)(y - 0),
\]
\[
z - 1 = 1(x - 0) + 0(y - 0) = x, \quad z = x + 1.
\]
Answer: \( z = x + 1 \).

26. (20 points) Find the linear approximation of \( f(x, y) = \sin(x + 2y) \) at \((0, 0)\), and use it to approximate \( f(0.1, 0.2) \). Using a calculator, compare the approximation with the exact value of \( f(0.1, 0.2) \).

Solution: We have
\[
\frac{\partial f}{\partial x} = \cos(x + 2y) \cdot 1, \quad \frac{\partial f}{\partial x}(0, 0) = 1 \cdot 1 = 1,
\]
\[
\frac{\partial f}{\partial y} = \cos(x + 2y) \cdot 2, \quad \frac{\partial f}{\partial y}(0, 0) = 1 \cdot 2 = 2,
\]
so the linear approximation has the form
\[
f(x, y) \approx z_0 + \frac{\partial f}{\partial x}(0, 0)(x - 0) + \frac{\partial f}{\partial y}(0, 0)(y - 0) = 0 + 1(x - 0) + 2(y - 0) = x + 2y.
\]
At \((x, y) = (-0.1, 0.2)\) we have \( f(x, y) \approx -0.1 + 2 \cdot 0.2 = 0.3 \). The actual value of \( f(-0.1, 0.2) \) equals
\[
f(-0.1, 0.2) = \sin(-0.1 + 2 \cdot 0.2) = \sin(0.3) \approx 0.2955.
\]