MAT 17C, Fall 2017
Solutions to Homework 2

Section 10.3: Find the partial derivatives:
12. (20 points) $f(x, y) = e^{-y^2} \cos(x^2 - y^2)$

Solution: By the Product Rule and the Chain Rule we have

$$\frac{\partial f}{\partial x} = 0 \cdot \cos(x^2 - y^2) + e^{-y^2} \cdot (-\sin(x^2 - y^2)) \cdot (2x) = -2xe^{-y^2} \sin(x^2 - y^2),$$

$$\frac{\partial f}{\partial y} = e^{-y^2} \cdot (-2y) \cdot \cos(x^2 - y^2) + e^{-y^2} \cdot (-\sin(x^2 - y^2)) \cdot (-2y) =$$

$$-2ye^{-y^2} \cos(x^2 - y^2) + 2ye^{-y^2} \sin(x^2 - y^2).$$

14. (20 points) $f(x, y) = \ln(3x^2 - xy)$.

Solution: By the Chain Rule we have

$$\frac{\partial f}{\partial x} = \frac{1}{3x^2 - xy} \cdot (6x - y), \frac{\partial f}{\partial y} = \frac{1}{3x^2 - xy} \cdot (-x).$$

30. (20 points) The prey density $H(t)$ and the predator density $P(t)$ are related by the equation

$$\frac{1}{H}\frac{dH}{dt} = r(1 - \frac{H}{k}) - aP,$$

where a, r, k are some positive constants. Investigate how an increase in (a) prey density and (b) predator density affects the per capita growth rate of this prey species.

Solution: The growth rate of prey species equals $\frac{dH}{dt}$, so per capita growth rate equals

$$\frac{1}{H}\frac{dH}{dt} = r(1 - \frac{H}{k}) - aP = r - rH/k - aP.$$

Let us denote the right hand side by $f(H, P)$, then

$$\frac{\partial f}{\partial H} = -r/k, \ \frac{\partial f}{\partial H} = -a.$$

Therefore the increases in prey density or in predator density both decrease the per capita growth rate $f(H, P)$.

Section 10.4: 8. (20 points) Find the tangent plane to the graph of $f(x, y) = e^x \cos y$ at the point $(0, 0, 1)$.

Solution: We have

$$\frac{\partial f}{\partial x} = e^x \cos y,\ \frac{\partial f}{\partial x}(0, 0) = 1 \cdot 1 = 1,$$

$$\frac{\partial f}{\partial y} = e^x(-\sin y),\ \frac{\partial f}{\partial y}(0, 0) = 1 \cdot 0 = 0,$$

so the equation of the tangent plane has the form

$$z - z_0 = \frac{\partial f}{\partial x}(0, 0)(x - 0) + \frac{\partial f}{\partial y}(0, 0)(y - 0),$$

$$z - 1 = 1(x - 0) + 0(y - 0) = x,\ z = x + 1.$$

Answer: $z = x + 1$.

26. (20 points) Find the linear approximation of $f(x, y) = \sin(x + 2y)$ at $(0, 0)$, and use it to approximate $f(0.1, 0.2)$. Using a calculator, compare the approximation with the exact value of $f(0.1, 0.2)$.

Solution: We have

$$\frac{\partial f}{\partial x} = \cos(x + 2y) \cdot 1,\ \frac{\partial f}{\partial x}(0, 0) = 1 \cdot 1 = 1,$$

$$\frac{\partial f}{\partial y} = \cos(x + 2y) \cdot 2,\ \frac{\partial f}{\partial y}(0, 0) = 1 \cdot 2 = 2,$$

so the linear approximation has the form

$$f(x, y) \approx z_0 + \frac{\partial f}{\partial x}(0, 0)(x - 0) + \frac{\partial f}{\partial y}(0, 0)(y - 0) = 0 + 1(x - 0) + 2(y - 0) = x + 2y.$$

At $(x, y) = (-0.1, 0.2)$ we have $f(x, y) \approx -0.1 + 2 \cdot 0.2 = 0.3$. The actual value of $f(-0.1, 0.2)$ equals

$$f(-0.1, 0.2) = \sin(-0.1 + 2 \cdot 0.2) = \sin(0.3) \approx 0.2955.$$