MAT 17C, Fall 2017
Solutions to homework 3

Chapter 10.5: 2. (20 points) Let $f(x,y) = e^x \sin y$ with $x(t) = t$ and $y(t) = t^3$. Find the derivative of $w = f(x,y)$ with respect to t when $t = 1$.

Solution: We have

$$\frac{\partial f}{\partial x} = e^x \sin y \frac{\partial f}{\partial y} = e^x \cos y,\ x'(t) = 1,\ y'(t) = 3t^2,$$

so

$$w'(t) = \frac{\partial f}{\partial x}x'(t) + \frac{\partial f}{\partial y}y'(t) = e^x \sin y \cdot 1 + e^x \cos y \cdot 3t^2 = e^t \sin(t^3) + 3e^t \cos(t^3) \cdot t^2.$$

At $t = 1$ we get

$$w'(1) = e \sin(1) + 3e \cos(1).$$

12. (20 points) . Find $\frac{dy}{dx}$ if $\cos(x^2 + y^2) = \sin(x^2 y^2)$.

Solution: We have

$$f(x,y) = \cos(x^2 + y^2) - \sin(x^2 y^2) = 0,$$

by Chain Rule

$$\frac{\partial f}{\partial x} = -2x \sin(x^2 + y^2) - 2x \cos(x^2 - y^2),$$
$$\frac{\partial f}{\partial y} = -2y \sin(x^2 + y^2) + 2y \cos(x^2 - y^2),$$

and

$$y'(x) = -\frac{\left(\frac{\partial f}{\partial x}\right)}{\left(\frac{\partial f}{\partial y}\right)} = -\frac{-2x \sin(x^2 + y^2) - 2x \cos(x^2 - y^2)}{-2y \sin(x^2 + y^2) + 2y \cos(x^2 - y^2)} =$$
$$\frac{x \sin(x^2 + y^2) + x \cos(x^2 - y^2)}{-y \sin(x^2 + y^2) + y \cos(x^2 - y^2)}.$$

30. (20 points) Find the directional derivative of $f(x,y) = ye^{x^2}$ at $(0,2)$ in direction $(4,-1)$.

Solution: We have

$$\frac{\partial f}{\partial x} = 2xye^{x^2},\ \frac{\partial f}{\partial y} = e^{x^2},$$

so

$$\frac{\partial f}{\partial x}(0,2) = 0,\ \frac{\partial f}{\partial y}(0,2) = 1,$$

so $\operatorname{grad} f(0,2) = (0,1)$. The directional derivative equals

$$(\operatorname{grad} f) \cdot (4,-1) = (0,1) \cdot (4,-1) = -1.$$

43. (20 points) Chemotaxis is the chemically directed movement of organisms up a concentration gradient - that is, in the direction in which the concentration increases most rapidly. The slime mold *Dictyostelium discoideum* exhibits this phenomenon. Single-celled amoebas of this species move up the concentration gradient of a chemical called cyclic adenosine monophosphate (AMP). Suppose the concentration of cyclic AMP at the point (x, y) in the xy plane is given by

$$f(x, y) = \frac{4}{|x| + |y| + 1}$$

If you place an amoeba at the point $(3, 1)$ in the xy plane, determine in which direction the amoeba will move if its movement is directed by chemotaxis.

Solution: The amoeba will move in the direction of the gradient of $f(x, y)$. Near $(3, 1)$ one has $|x| = x, |y| = y$, so

$$f(x, y) = \frac{4}{x + y + 1} = 4(x + y + 1)^{-1}, \ \frac{\partial f}{\partial x} = -4(x + y + 1)^{-2} = \frac{\partial f}{\partial y}.$$

Therefore

$$\operatorname{grad} f(3, 1) = (-4/25, -4/25).$$

44. (20 points) Suppose an organism moves down a sloped surface along the steepest line of descent. If the surface is given by $f(x, y) = x^2y^2$, find the direction in which the organism will move at the point $(2, 3)$.

Solution: We have

$$\frac{\partial f}{\partial x} = 2x, \ \frac{\partial f}{\partial y} = -2y,$$

so

$$\operatorname{grad} f(2, 3) = (4, -6).$$

The steepest line of descent is directed oppositely to the gradient, so the organism moves in direction

$$-\operatorname{grad} f(2, 3) = (-4, 6).$$