MAT 17C, Fall 2017
Solutions to homework 4

Chapter 9.3: Find the eigenvalues λ_1, λ_2 and the eigenvectors v_1, v_2. Draw the vectors v_1, v_2, Av_1, Av_2.

50. (25 points) $A = \begin{pmatrix} 0 & 0 \\ 1 & -3 \end{pmatrix}$.

Solution: The characteristic equation has the form:

$$0 = \det \begin{pmatrix} 0-\lambda & 0 \\ 1 & -3-\lambda \end{pmatrix} = -\lambda(-3-\lambda),$$

so the eigenvalues are $\lambda_1 = 0$ and $\lambda_2 = -3$. The coordinates of the first eigenvector satisfy

$$0x + 0y = 0, \ x - 3y = 0,$$

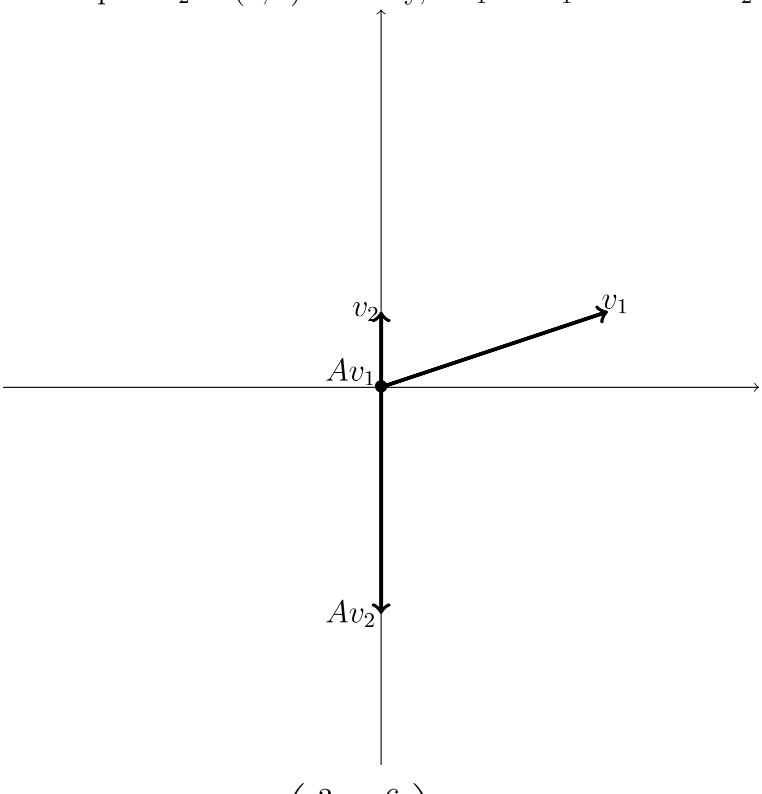
so we can pick $v_1 = (3, 1)$. The coordinates of the second eigenvector satisfy

$$\begin{pmatrix} 0-(-3) & 0 \\ 1 & -3-(-3) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix},$$

so

$$3x + 0y = 0, \ x + 0y = 0,$$

so we can pick $v_2 = (0, 1)$. Finally, $Av_1 = 0v_1 = 0$ and $Av_2 = -3v_2 = (0, -3)$.



54. (25 points) $A = \begin{pmatrix} 3 & 6 \\ -1 & -4 \end{pmatrix}$.

Solution: The characteristic equation has the form:

$$0 = \det \begin{pmatrix} 3-\lambda & 6 \\ -1 & -4-\lambda \end{pmatrix} = (3-\lambda)(-4-\lambda)+6 =$$

$$-12-3\lambda+4\lambda+\lambda^2+6 = \lambda^2+\lambda-6 = (\lambda-2)(\lambda+3).$$

so the eigenvalues are $\lambda_1 = 2$ and $\lambda_2 = -3$. The coordinates of the first eigenvector satisfy

$$\begin{pmatrix} 3-2 & 6 \\ -1 & -4-2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix},$$

so

$$x+6y = 0, \ -x-6y = 0,$$

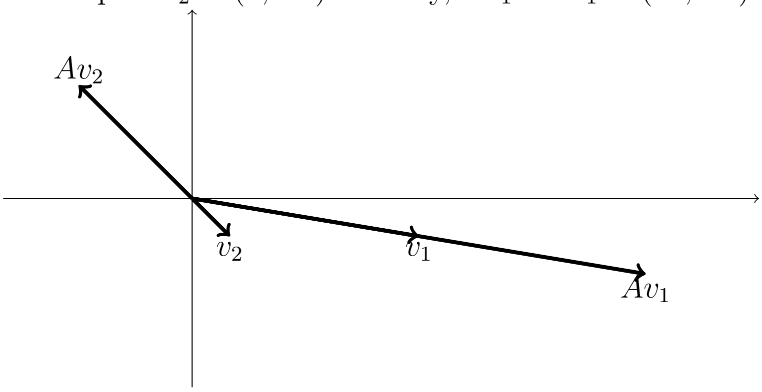
so we can pick $v_1 = (6, -1)$ The coordinates of the second eigenvector satisfy

$$\begin{pmatrix} 3-(-3) & 6 \\ -1 & -4-(-3) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix},$$

so

$$6x+6y = 0, \ -x-y = 0,$$

so we can pick $v_2 = (1, -1)$. Finally, $Av_1 = 2v_1 = (12, -2)$ and $Av_2 = -3v_2 = (-3, 3)$.



60. (25 points) Find the eigenvalues for the matrix $A = \begin{pmatrix} -1 & 4 \\ 0 & -2 \end{pmatrix}$.

Solution: We have

$$0 = \det \begin{pmatrix} -1-\lambda & 4 \\ 0 & -2-\lambda \end{pmatrix} = (-1-\lambda)(-2-\lambda),$$

so the eigenvalues are equal to $\lambda_1 = -1, \lambda_2 = -2$.

Chapter 10.6: 4. (25 points) Find the critical points for the function $f(x, y) = xy - 2y^2$ and determine their type.

Solution: We have $f_x = y$, $f_y = x-4y$, so the critical points are given by the equation

$$y = x-4y = 0 \ \Rightarrow \ x = y = 0.$$

Therefore there is only one critical point $(0,0)$. The Hessian of $f(x,y)$ has the form

$$H = \begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & -4 \end{pmatrix}$$

The characteristic equation has the form

$$0 = \det \begin{pmatrix} 0-\lambda & 1 \\ 1 & -4-\lambda \end{pmatrix} = -\lambda(-4-\lambda) - 1 = \lambda^2 + 4\lambda - 1.$$

By quadratic formula, we have

$$\lambda_{1,2} = \frac{-4 \pm \sqrt{4^2 + 4 \cdot 1}}{2} = \frac{-4 \pm \sqrt{20}}{2}.$$

Since $\lambda_1 = \frac{-4+\sqrt{20}}{2} > 0$ and $\lambda_2 = \frac{-4-\sqrt{20}}{2} < 0$, this is a saddle.