Math 21A, Sections F01-F02
Practice problems for Midterm 1

This practice sheet contains more problems than the actual exam.

1. Consider the functions:

$$f(x) = \frac{x^2+1}{x^2-1},\ g(x) = ||x+1|-1|,\ h(x) = \frac{1}{\ln x}.$$

For each of them:
a) Find the domain
b) Determine the intersection points of the graph with the coordinate axis
c) Find all vertical and horizontal asymptotes (if any)
d) Sketch the graph

2. Find all values of the parameters a and b such that the function $f(x) = ax + b$ is (a) even; (b) odd.

3. Find all values of the parameter c such that the function

$$f(x) = \begin{cases} \sin x, & \text{if } x < c, \\ \cos x, & \text{if } x \geq c \end{cases}$$

is continuous.

4. Compute the limits:

a)

$$\lim_{x\to 0}(\sin^3 x - \cos x)$$

b)

$$\lim_{x\to 0}\frac{\sqrt{x+1}}{\sqrt{x}}$$

c)

$$\lim_{x\to 3}\frac{x^2-2x-3}{x^2-3x}.$$

d)

$$\lim_{x\to 0}\frac{7x^3+8x^2+9x}{11x+12x^2-13x^3}.$$

e)

$$\lim_{x\to +\infty} e^{-x^2}$$

f)

$$\lim_{x\to+\infty} (\ln(x+1) - \ln x)$$

g)

$$\lim_{x\to+\infty} \frac{7x^2+8x+9}{11+12x-13x^2}.$$

h)

$$\lim_{x\to 2^+} \arctan(\frac{1}{x-2}).$$

i)

$$\lim_{x\to 0} x^2\cos(1/x).$$

j)

$$\lim_{x\to+\infty} \frac{2x^3+\sqrt{x^4+7}}{3x^3-5}$$

5. Find $\lim_{x\to 1} f(x)$, where

$$f(x) = \begin{cases} x^2+3x+4, & \text{if } x \geq 1, \\ 7x^2+1, & \text{if } x < 1. \end{cases}$$

6. Plot the graph of the function

$$\mathrm{sgn}(x) = \begin{cases} \frac{|x|}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0. \end{cases}$$

Is it continuous?

7. For the function

$$f(x) = \begin{cases} x^2+x-9, & \text{if } |x| \geq 3, \\ x, & \text{if } |x| < 3, \end{cases}$$

find one-sided limits at $x=-3$, $x=0$, $x=3$. Is this function continuous?

8. Use the Intermediate Function Theorem to show that the equation

$$x^3+x-3=0$$

has a solution on the interval $[1,2]$.