

MAT 21A, practice problems for Midterm 2

- Find the derivatives of the following functions:
 - $f(x) = \frac{\sin x}{\ln x}$
 - $f(x) = xe^{\cos x}$
 - $f(x) = e^{\ln(2+x) - \ln(1+x)}$.
 - * $f(x) = (\sin x)^{\cos x}$
 - $\sqrt{\frac{x-1}{x+1}}$.
- Find the derivative of $y(x)$ using implicit differentiation, if
 - $3x^2 + 2y^2 = 10$
 - $\cos(x) + \cos(y) = 15$
 - $\frac{x}{y} - \frac{y}{x} = 1$
- Find the equation of the tangent line to the graph of $f(x) = x^4e^{-x}$ at a point $(1, e^{-1})$.
- Find the maximal and minimal values of a given function on a given interval:
 - $f(x) = x + \sin x$, $[0, 4]$
 - $f(x) = x^3 - 27x + 1$, $[-5, 5]$
 - $\frac{\ln x}{x}$, $[1, 2]$.
- For a given function, determine the intervals where it is increasing/decreasing, and find points of maximum/minimum.
 - $f(x) = 2\sqrt{x} - x$
 - $f(x) = \frac{e^x}{x^2}$
 - $f(x) = x^4 - 4x$
 - $f(x) = \ln(x^2 - 3x + 2)$.
- (ex. 11 on p.270) You are designing a rectangular poster to contain 50 square inches of printing with a 4-in. margin at the top and bottom and 2-in. margin at each side. What overall dimensions will minimize the amount of paper used?

7. A ship is moving east with speed 1, another ship is moving north with speed 2. At some moment, their coordinates were $(-1, 0)$ and $(0, -1)$, respectively. What will be the minimal distance between these ships?
8. (ex. 58 on p. 275) The 800-room Mega Motel chain is filled to capacity when the room charge is \$50 per night. For each \$10 increase in the room charge, 40 fewer rooms are filled each night. What charge per room will result in the maximum revenue per night?