MAT 21A, practice problems for the final exam

1. Compute the limits:

a) $\lim_{x\to 3} e^{1/x}$

b) $\lim_{x\to 3} \ln(x-3)$

c) $\lim_{x\to 3} \frac{\ln(x-2)}{x-3}$

d) $\lim_{x\to\infty} \frac{8x^5-7x^3+9}{(3x^2-1)(2x^3-3)}$

e) $\lim_{x\to\infty} \frac{e^x}{x^3}$

f) $\lim_{x\to 0} \frac{\arctan(x)-x}{x^3}$

2. Compute the derivatives of the following functions:

a) $f(x) = x\ln x - x$

b) $f(x) = e^{3x^2}$

c) $f(x) = (x-1)^5 \cos x$

d) $f(x) = \sin(\ln x)$

e) $f(x) = \frac{e^{3x+2}}{\cos x+2}$

3. Find the minimal and maximal values of a function:

a) $f(x) = x^2 e^{-x}$ on $[0,1]$

b) $f(x) = 2x^3 - 3x^2 + 1$ on $[-1,2]$

c) $f(x) = \sin^2 x$ on $[0,\pi]$

d) $\frac{x}{1+x^2}$ on $[-2,2]$

4. Find the equation of the tangent line to the graph of $f(x) = \ln x$ at $x = 5$.

5. For a given function:

- Find the domain
- Determine the equations of vertical and horizontal asymptotes
- Find the derivative and determine the intervals where the function is increasing/decreasing

- Find the second derivative and determine the intervals where the function is concave up/down, find inflection points
- Draw the graph using all the information above

a) $f(x) = 2x^3 - 3x^2 + 1$

b) $f(x) = xe^{-x}$

c) $f(x) = \ln(x^2 + 1)$

d) $f(x) = \frac{x-1}{x+1}$

6. Consider the function

$$f(x) = \begin{cases} x + 1, & \text{if } x < -1 \\ x^2 + ax + b, & \text{if } x \geq -1. \end{cases}$$

a) For which values of the parameters it is continuous?

b) For which values of the parameters it has a derivative at every point?

7. Consider the curve given by the equation $x^{2/3} + y^{2/3} = 1$. Find y' using implicit differentiation and sketch this curve. (Note: assume that $x^{2/3} = \sqrt[3]{x^2}$ is defined for all x).

8. Consider the curve given by the equation $y^2 = x^3 - x$. Find y' using implicit differentiation and sketch this curve.

9. An open rectangular box with square base is to be made from 1 area unit of material. What dimensions will result in a box with the largest possible volume ?

10. A TV set costs 100. If its price is lowered by $a\%$, the sales would increase by $2a\%$. Find the discount amount a which yields the maximal profit.