Section 1.2: 38. (10 points) Graph the function $y = |1 - x| - 1$.

**Solution 1:** We have $|1 - x| = 1 - x$, if $1 - x \geq 0$ (that is, $x \leq 1$) and $|1 - x| = x - 1$ if $x > 1$. Therefore

$$y(x) = \begin{cases} 
-x & \text{if } x \leq 1, \\
2 - x & \text{if } x > 1.
\end{cases}$$

See the graph below.

**Solution 2:** The graph of the linear function $y = 1 - x$ has the form:

The graph of $y = |1 - x|$ is obtained from it by reflection of its negative parts in the horizontal axis:

The graph of $y = |1 - x| - 1$ is obtained from it by vertical shift down by 1:
54. (10 points) Sketch the graph of the function \( y = \frac{1}{(x+1)^2} \).

**Solution:** The graph of \( y = 1/x^2 \) has the form:

The graph of \( y = 1/(1 + x)^2 \) is obtained from it by the shift left by 1:
Section 1.3: 22. (10 points) Sketch the graph of the function $y = \cos \left( x + \frac{2\pi}{3} \right) - 2$.

Solution: The graph of $y = \cos x$ has the form:

$$
\begin{array}{c}
\includegraphics[width=0.5\textwidth]{graph1.png}
\end{array}
$$

The graph of $y = \cos \left( x + \frac{2\pi}{3} \right)$ is obtained from it by the horizontal shift by $\left( -\frac{2\pi}{3} \right)$:

$$
\begin{array}{c}
\includegraphics[width=0.5\textwidth]{graph2.png}
\end{array}
$$

The graph of $y = \cos \left( x + \frac{2\pi}{3} \right) - 2$ is obtained from it by the vertical shift by $(-2)$:
Section 1.6: 32. (10 points) Consider the function $f(x) = \sqrt[3]{x^3 - 3}$. Find the inverse function $f^{-1}(x)$ and identify the domain and range of $f^{-1}$.

Solution: Let us solve the equation $y = \frac{\sqrt{x}}{\sqrt{x - 3}}$ for $x$. We have:

$$\sqrt{x} = y(\sqrt{x} - 3) = y\sqrt{x} - 3y, \quad \sqrt{x} - y\sqrt{x} = \sqrt{x}(1 - y) = -3y,$$

so

$$\sqrt{x} = \frac{-3y}{1 - y}.$$

This equation has a solution, if $y \neq 1$ and $\frac{-3y}{1 - y} \geq 0$, and the solution is given by the equation

$$x = f^{-1}(y) = \left(\frac{-3y}{1 - y}\right)^2.$$

To solve the inequality $\frac{-3y}{1 - y} \geq 0$, consider 3 cases:

a) If $y > 1$ then $-3y < 0, 1 - y < 0$ and the fraction is positive.

b) If $0 \leq y < 1$ then $-3y \leq 0, 1 - y > 0$ and the fraction is negative.

c) If $y < 0$ then $-3y > 0, 1 - y > 0$ and the fraction is positive.

Therefore the domain of $f^{-1}$ is $(-\infty, 0] \cup (1, +\infty)$. Finally, $f(x)$ is defined if $x \geq 0$ and $\sqrt{x} \neq 3$, so the domain of $f$ is $[0, 9) \cup (9, +\infty)$. The range of $f^{-1}$ coincides with the domain of $f$.

Answer: $f^{-1}(y) = \left(\frac{-3y}{1 - y}\right)^2$, domain of $f^{-1}$ equals $(-\infty, 0] \cup (1, +\infty)$ and the range of $f^{-1}$ equals $[0, 9) \cup (9, +\infty)$. 