MATH 21B, practice problems for Midterm 2

This practice sheet contains more problems than the actual exam.

1. Consider the function $f(x) = \frac{1}{2}(e^x + e^{-x})$.

a) Find the length of the curve given by the equation $y = f(x)$, $-1 \le x \le 1$.

b) Let R be the region bounded by the graph of $f(x)$ and the lines $x = 1, x = -1$ and $y = 0$. Find the area of R.

c) Find the coordinates of the center of mass of R.

d) Consider the solid obtained by rotation of R about the x-axis. Find its volume and surface area.

e)* Consider the solid obtained by rotation of R about the y-axis. Find its volume and surface area.

2. A submarine has a shape of a round cylinder with radius R and length h. It is fully submerged such that its axis is horizontal at depth d (and $d > R$). Find the total force of water pressure acting on the front of the submarine.

3. By Newton's law of gravity two bodies with masses m and M are attracted to each other with a force

$$F = G\frac{mM}{d^2},\ G \text{ is a constant},$$

where d is the distance between their centers. A planet has a shape of a round ball of radius R and has mass M, a rocket of mass m starts vertically from its surface.

a) Compute the work $W(H)$ needed to move the rocket to a height H above the surface. What happens at the limit $H \to \infty$?

b) The *escape velocity* is the minimum speed needed for the rocket to move infinitely away from the planet without using its engine. Alternatively, if v is the escape velocity then the kinetic energy $mv^2/2$ of the rocket at start equals $W(\infty)$. Use this idea to find the escape velocity.

4. A *cardioid* is given by the equations:

$$x(t) = 2\cos(t) - \cos(2t), y(t) = 2\sin(t) - \sin(2t).$$

Compute the total length of the cardioid. *Hint: you may need a trigonometric identity:*

$$\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b).$$

5. Solve the differential equations:

a) $y' = y/x$

b) $y' = e^{x+y}$

6. A sample of some radioactive material reduced to 90% of the original mass in a year. Find the half-life time for this material.