MAT 21B, Fall 2016
Solutions to Homework 1

Section 4.8: Solve the initial value problems:
95.(10 points) $\frac{dy}{dx} = 3x^{-2/3},\ y(-1) = -5.$

Solution:

$$y(x) = \int 3x^{-2/3}dx = 3\int x^{-2/3}dx = 3\frac{x^{1/3}}{1/3} + C = 9\sqrt[3]{x} + C.$$

Now $y(-1) = -9 + C = -5$, so $C = 4$ and $y(x) = 9\sqrt[3]{x} + 4$.

106.(10 points) $\frac{d^y}{dx^2} = 0, y'(0) = 2, y(0) = 0.$

Solution: Since $y''(x) = 0$, $y'(x)$ is a constant function. Since $y'(0) = 2$, we conclude $y'(x) = 2$, so $y(x) = 2x + C$. Since $y(0) = 0$, $C = 0$ and $y(x) = 2x$.

Section 5.1: 8.(10 points) Estimate the are under the graph of $f(x) = 4 - x^2$ on $[-2, 2]$ using midpoints and (a) two (b) four rectangles.

Solution:(a) For two rectangles, we divide $[-2, 2]$ into segments $[-2, 0]$ and $[0, 2]$ and choose midpoints $x_1 = -1$ and $x_2 = 1$ on them. The integral sum equals $(f(-1) + f(1)) \cdot 2 = (3 + 3) \cdot 2 = 12$.

(b) For four rectangles, we divide $[-2, 2]$ into segments $[-2, -1], [-1, 0], [0, 1]$ and $[1, 2]$ and choose midpoints $\pm 0.5, \pm 1.5$ on them. The integral sum equals

$$(f(-1.5) + f(-0.5) + f(0.5) + f(1.5)) \cdot 1 = 1.75 + 3.75 + 3.75 + 1.75 = 11.$$