MAT 21B, Fall 2016
Solutions to Homework Assignment 2

Section 5.4: Compute the following integrals:
4.(10 points) $\int_{-1}^{1} x^{299} dx$.

Answer: 0.

Solution:

$$\int_{-1}^{1} x^{299} dx = \frac{x^{300}}{300}|_{-1}^{1} = \frac{1}{300} - \frac{1}{300} = 0.$$

20.(10 points) $\int_{-\sqrt{3}}^{\sqrt{3}} (t+1)(t^2+4)dt$

Answer: $10\sqrt{3}$.

Solution:

$$\int (t+1)(t^2+4)dt = \int (t^3+t^2+4t+4)dt = \frac{t^4}{4} + \frac{t^3}{3} + 2t^2 + 4t + C,$$

therefore

$$\int_{-\sqrt{3}}^{\sqrt{3}} (t+1)(t^2+4)dt == \left(\frac{9}{4} + \frac{3\sqrt{3}}{3} + 6 + 4\sqrt{3}\right) - \left(\frac{9}{4} - \frac{3\sqrt{3}}{3} + 6 - 4\sqrt{3}\right) =$$

$$\left(8\frac{1}{4} + 5\sqrt{3}\right) - \left(8\frac{1}{4} - 5\sqrt{3}\right) = 10\sqrt{3}.$$

24.(10 points) $\int_{1}^{8} \frac{(x^{1/3}+1)(2-x^{2/3})}{x^{1/3}} dx$

Answer: $-137/20$.

Solution:

$$\int \frac{(x^{1/3}+1)(2-x^{2/3})}{x^{1/3}} dx = \int \frac{2x^{1/3}+2-x-x^{2/3}}{x^{1/3}} dx =$$

$$\int (2+2x^{-1/3}-x^{2/3}-x^{1/3})dx = 2x + \frac{2x^{2/3}}{2/3} - \frac{x^{5/3}}{5/3} - \frac{x^{4/3}}{4/3} + C =$$

$$2x + 3x^{2/3} - \frac{3x^{5/3}}{5} - \frac{3x^{4/3}}{4} + C,$$

therefore

$$\int_{1}^{8} \frac{(x^{1/3}+1)(2-x^{2/3})}{x^{1/3}} = \left(2\cdot 8 + 3\cdot 4 - \frac{3\cdot 32}{5} - \frac{3\cdot 16}{4}\right) - \left(2 + 3 - \frac{3}{5} - \frac{3}{4}\right) =$$

$$(16+12-96/5-12) - (5-3/5-3/4) = 11 - 93/5 + 3/4 = -137/20.$$

30.(10 points) $\int_{1}^{2} \left(\frac{1}{x} - e^{-x}\right) dx$.
Answer: $\ln 2 + e^{-2} - e^{-1}$.

Solution:

$$\int \left(\frac{1}{x} - e^{-x}\right) dx = \ln|x| + e^{-x} + C,$$

therefore

$$\int_{1}^{2} \left(\frac{1}{x} - e^{-x}\right) dx = (\ln 2 + e^{-2}) - (0 + e^{-1}) = \ln 2 + e^{-2} - e^{-1}.$$