MAT 21B, Fall 2016
Solutions to Homework Assignment 6

Section 7.2: Solve the differential equations:
10. (10 points) $dy/dx = x^2\sqrt{y}$.

Solution: We have $\frac{dy}{dx} = x^2\sqrt{y}$, so

$$\int \frac{dy}{\sqrt{y}} = \int x^2 dx,\ 2\sqrt{y} = \frac{x^3}{3} + C,\ \sqrt{y} = \frac{x^3}{6} + \frac{C}{2},$$

and $y = (\frac{x^3}{6} + \frac{C}{2})^2$.

20. (10 points) $dy/dx = xy + 3x - 2y - 6$.

Solution: We have

$$\frac{dy}{dx} = xy + 3x - 2y - 6 = (x-2)(y+3),$$

so

$$\int \frac{dy}{y+3} = \int (x-2)dx,\ \ln|y+3| = \frac{x^2}{2} - 2x + C,$$

$$y + 3 = \pm e^{\frac{x^2}{2} - 2x + C} = \pm e^C \cdot e^{\frac{x^2}{2} - 2x} = Ae^{\frac{x^2}{2} - 2x},$$

and

$$y(x) = Ae^{\frac{x^2}{2} - 2x} - 3.$$

Section 8.1: Compute the following integrals:
6. (10 points) $\int \frac{dx}{x - \sqrt{x}}$

Solution: Let $u = \sqrt{x}$, then $x = u^2$ and $dx = 2udu$. Therefore

$$\int \frac{dx}{x - \sqrt{x}} = \int \frac{2udu}{u^2 - u} = \int \frac{2du}{u-1} = 2\ln|u-1| + C = 2\ln|\sqrt{x} - 1| + C.$$

10. (10 points) $\int_1^2 \frac{8dx}{x^2 - 2x + 2}$.

Solution: We have

$$\int_1^2 \frac{8dx}{x^2 - 2x + 2} = \int_1^2 \frac{8dx}{(x-1)^2 + 1}.$$

Let $u = x - 1$, then $du = dx$ and the integral can be transformed to

$$\int_0^1 \frac{8du}{u^2 + 1} = 8\arctan(u)|_0^1 = 8\arctan(1) = 8 \cdot \frac{\pi}{4} = 2\pi.$$