MAT 21B, Fall 2016
Solutions to Homework Assignment 7

Compute the integrals:
Section 8.2: 46.(10 points) $\int \sqrt{x}e^{\sqrt{x}}dx$.

Solution: Let $u = \sqrt{x}$, then $x = u^2$ and $dx = 2udu$, so

$$\int \sqrt{x}e^{\sqrt{x}}dx = \int ue^u \cdot 2udu = 2\int u^2e^u du.$$

Integrate by parts:

$$2\int u^2e^u du = 2\int u^2 d(e^u) = 2u^2e^u - 2\int e^u d(u^2) = 2u^2e^u - 2\int e^u \cdot 2udu =$$

$$2u^2e^u - 4\int ue^u du.$$

Integrate by parts again:

$$4\int ue^u du = 4\int ud(e^u) = 4ue^u - 4\int e^u du = 4ue^u - 4e^u + C.$$

Therefore the integral equals:

$$2u^2e^u - 4ue^u + 4e^u - C = 2xe^{\sqrt{x}} - 4\sqrt{x}e^{\sqrt{x}} + 4e^{\sqrt{x}} - C.$$

48.(10 points) $\int_0^{\pi/2} x^3 \cos(2x)dx$

Solution: Integrate by parts:

$$\int x^3\cos(2x)dx = \frac{1}{2}\int x^3 d(\sin 2x) = \frac{1}{2}x^3\sin(2x) - \frac{1}{2}\int \sin(2x)d(x^3) =$$

$$\frac{1}{2}x^3\sin(2x) - \frac{3}{2}\int x^2\sin(2x)dx.$$

Integrate by parts again:

$$-\frac{3}{2}\int x^2\sin(2x)dx = \frac{3}{4}\int x^2 d(\cos(2x)) = \frac{3}{4}x^2\cos(2x) - \frac{3}{4}\int \cos(2x)d(x^2) =$$

$$\frac{3}{4}x^2\cos(2x) - \frac{3}{2}\int x\cos(2x)dx.$$

And again:

$$-\frac{3}{2}\int x\cos(2x)dx = -\frac{3}{4}\int xd(\sin 2x) = -\frac{3}{4}x\sin(2x) + \frac{3}{4}\int \sin(2x)dx =$$

$$-\frac{3}{4}x\sin(2x) - \frac{3}{8}\cos(2x)dx + C.$$

To sum up,

$$\int x^3\cos(2x)dx = \frac{1}{2}x^3\sin(2x) + \frac{3}{4}x^2\cos(2x) - \frac{3}{4}x\sin(2x) - \frac{3}{8}\cos(2x)dx + C.$$

Now $\sin(0) = \sin(2\cdot\frac{\pi}{2}) = \sin(\pi) = 0$, $\cos(0) = 1$ and $\cos(2\cdot\frac{\pi}{2}) = \cos(\pi) = -1$, so

$$\int_0^{\pi/2} x^3\cos(2x)dx = (0 - \frac{3}{4}\cdot\frac{\pi^2}{4} + \frac{3}{8}) - (0 + 0 - 0 - \frac{3}{8}) = \frac{3}{4}(1 - \frac{\pi^2}{4}).$$

Section 8.3: 16.(10 points) $\int 7\cos^7(t)dt$.

Solution:

$$\int 7\cos^7(t)dt = \int 7(\cos^2(t))^3\cos(t)dt = \int 7(1-\sin^2(t))^3\cos(t)dt.$$

Let $u = \sin(t)$, then $du = \cos t dt$, and

$$\int 7(1-\sin^2(t))^3\cos(t)dt = 7\int(1-u^2)^3du = 7\int(1-3u^2+3u^4-u^6)du =$$

$$7\left(u - u^3 + \frac{3}{5}u^5 - \frac{1}{7}u^7\right) + C = 7\left(\sin(t) - \sin^3(t) + \frac{3}{5}\sin^5(t) - \frac{1}{7}\sin^7(t)\right) + C.$$

56.(10 points) $\int_{-\pi/2}^{\pi/2}\cos(x)\cos(7x)dx$

Solution: We have

$$\cos(x)\cos(7x) = \frac{1}{2}(\cos(8x)+\cos(6x)),$$

so

$$\int_{-\pi/2}^{\pi/2}\cos(x)\cos(7x)dx = \frac{1}{2}\int_{-\pi/2}^{\pi/2}(\cos(8x)+\cos(6x))dx = \frac{1}{2}(\frac{1}{8}\sin(8x)+\frac{1}{6}\sin(6x))|_{-\pi/2}^{\pi/2} = 0.$$