MATH 21B, practice problems for the final exam

This practice sheet contains more problems than the actual exam.

1. Consider the region R bounded by the graph of $y = \sin(x)$ on $[0, \pi]$ and the x-axis.

a) Find the area of R

b) Find the volume of the solid of revolution obtained by rotation of R about the x-axis

c) Find the volume of the solid of revolution obtained by rotation of R about the y-axis

d) Find the coordinates of the center of mass of R.

2. Solve the following differential equations:

a) $y' = y$

b) $y' = \frac{1}{y}$

c) $y' = \frac{x}{y}$, $y(0) = 1$.

3. Compute the following integrals:

a) $\int \frac{3x+5}{x^2+2x} dx$

b) $\int \frac{3x+5}{x^2+2x+2} dx$

c) $\int \sin(3x) \cos(5x) dx$

d) $\int x\sqrt{4 - x^2} dx$

e) $\int x \ln x \, dx$

4. Compute the following definite integrals:

a) $\int_2^3 \frac{3x+5}{x^2+2x+1} dx$

b) $\int_0^\pi \sin^3 x \cos^5 x \, dx$

c) $\int_2^3 \frac{dx}{x \ln^2 x}$

d) $\int_0^1 \frac{x dx}{\sqrt{4-x^2}}$

e) $\int_0^\pi x \sin x \, dx$.

5. For the following indefinite integrals, determine if they converge or diverge. If they converge, find their values.

a) $\int_0^1 \frac{dx}{\sqrt{x}}$

b) $\int_1^\infty \frac{dx}{\sqrt{x}}$

c) $\int_0^\infty x^2 e^{-x} dx$

d) $\int_0^\infty \frac{dx}{x^2+3x+2}$

e) $\int_0^\infty \frac{x dx}{x^2+3x+2}$

6. Find the lengths of the following curves:

a) $x = 3\cos t + \cos(3t)$, $y = 3\sin t + \sin(3t)$, $0 \le t \le 2\pi$.

b)* $y = x^2$, $1 \le x \le 2$.

7. A gate in a reservoir has a shape of a half-circle of radius R with the center at water level. Find the total force of water pressure on this gate, if the water density equals ρ and the gravity acceleration equals g.

8. Determine the surface area of the solid obtained by rotating $y = \sqrt[3]{x}$, $1 \le x \le 2$, about the y-axis.

9**. Compute the integral $\int \frac{dx}{x^4+1}$.