

MATH 21B, more practice problems for Midterm 2

1. Consider the function $f(x) = x^{10}$ on the interval $[0, 1]$.
 - a) Find the volume obtained by rotation of the region below the graph around x -axis.
 - b) Find the volume obtained by rotation of the region below the graph around y -axis.
 - c) Find the center of mass of the region below the graph of $f(x)$.
2. Consider the function $f(x) = \frac{x^3}{4} + \frac{1}{3x}$ on the interval $[1, 2]$.
 - a) Find the length of this graph.
 - b) Find the area of the surface obtained by rotation of this graph around x -axis.
3. A spherical tank of radius 1 is full of water. Find the amount of work necessary to pump out all the water through an opening at the top of the tank.
4. Solve the differential equations:
 - a) $y' = (1 + x^2)(1 + y^2)$
 - b) $y' = \frac{\sin x}{\sin y}$.