MATH 21B, more practice problems for Midterm 2

1. Consider the function $f(x) = x^{10}$ on the interval $[0, 1]$.
   
a) Find the volume obtained by rotation of the region below the graph around $x$-axis.
   
b) Find the volume obtained by rotation of the region below the graph around $y$-axis.
   
c) Find the center of mass of the region below the graph of $f(x)$.

2. Consider the function $f(x) = x^3 + \frac{1}{3x}$ on the interval $[1, 2]$.
   
a) Find the length of this graph.
   
b) Find the area of the surface obtained by rotation of this graph around $x$-axis.

3. A spherical tank of radius 1 is full of water. Find the amount of work necessary to pump out all the water through an opening at the top of the tank.

4. Solve the differential equations:
   
a) $y' = (1 + x^2)(1 + y^2)$
   
b) $y' = \frac{\sin x}{\sin y}$. 