MATH 21B, practice problems for Midterm 2

This practice sheet contains more problems than the actual exam.

1. Consider the function $f(x) = \frac{1}{2}(e^x + e^{-x})$.

a) Find the length of the curve given by the equation $y = f(x)$, $-1 \le x \le 1$.

b) Let R be the region bounded by the graph of $f(x)$ and the lines $x = 1, x = -1$ and $y = 0$. Find the area of R.

c) Find the coordinates of the center of mass of R.

d) Consider the solid obtained by rotation of R about the x-axis. Find its volume and surface area.

e) Consider the solid obtained by rotation of R about the y-axis. Find its volume.

2. A submarine has a shape of a round cylinder with radius R and length h. It is fully submerged such that its axis is horizontal at depth d (and $d > R$). Find the total force of water pressure acting on the front of the submarine.

3. By Newton's law of gravity two bodies with masses m and M are attracted to each other with a force

$$F = G\frac{mM}{d^2}, \ G \text{ is a constant,}$$

where d is the distance between their centers. A planet has a shape of a round ball of radius R and has mass M, a rocket of mass m starts vertically from its surface.

a) Compute the work $W(H)$ needed to move the rocket to a height H above the surface. What happens at the limit $H \to \infty$?

b) The *escape velocity* is the minimum speed needed for the rocket to move infinitely away from the planet without using its engine. Alternatively, if v is the escape velocity then the kinetic energy $mv^2/2$ of the rocket at start equals $W(\infty)$. Use this idea to find the escape velocity.

4. A plate is bounded by the parabola $y = x^2$ and the line $y = 1$. Find its center of mass.

5. Solve the differential equations:
a) $y' = y/x$
b) $y' = e^{x+y}$