

## MATH 21B, practice problems for the final exam

*This practice sheet contains more problems than the actual exam.*

- Consider the region  $R$  bounded by the graph of  $y = \sin(x)$  on  $[0, \pi]$  and the  $x$ -axis.
  - Find the area of  $R$
  - Find the volume of the solid of revolution obtained by rotation of  $R$  about the  $x$ -axis
  - Find the volume of the solid of revolution obtained by rotation of  $R$  about the  $y$ -axis
  - Find the coordinates of the center of mass of  $R$ .
- Solve the following differential equations:
  - $y' = y$
  - $y' = \frac{1}{y}$
  - $y' = \frac{x}{y}$ ,  $y(0) = 1$ .
- Compute the following integrals:
  - $\int \frac{3x+5}{x^2+2x} dx$
  - $\int \frac{3x+5}{x^2+2x+2} dx$
  - $\int \sin(3x) \cos(5x) dx$
  - $\int x\sqrt{4-x^2} dx$
  - $\int x \ln x dx$
- Compute the following definite integrals:
  - $\int_2^3 \frac{3x+5}{x^2+2x+1} dx$
  - $\int_0^\pi \sin^3 x \cos^5 x dx$
  - $\int_2^3 \frac{dx}{x \ln^2 x}$
  - $\int_0^1 \frac{x dx}{\sqrt{4-x^2}}$
  - $\int_0^\pi x \sin x dx$ .
- Find the lengths of the following curves:
  - $y = \frac{1}{2}x^2$ ,  $1 \leq x \leq 2$ .
  - \*  $x = 3 \cos t + \cos(3t)$ ,  $y = 3 \sin t + \sin(3t)$ ,  $0 \leq t \leq 2\pi$ .
- A gate in a reservoir has a shape of a half-circle of radius  $R$  with the center at water level. Find the total force of water pressure on this gate, if the water density equals  $\rho$  and the gravity acceleration equals  $g$ .
- Determine the surface area of the solid obtained by rotating  $y = x^3$ ,  $1 \leq x \leq 2$ , about the  $x$ -axis.
- 8\*\*. Compute the integral  $\int \frac{dx}{(x+1)(x+2)(x+3)}$ .