

MATH 21B, answers to practice problems for the final exam

This practice sheet contains more problems than the actual exam.

1. Consider the region R bounded by the graph of $y = \sin(x)$ on $[0, \pi]$ and the x -axis.

a) Find the area of R . **Answer:** 2.

b) Find the volume of the solid of revolution obtained by rotation of R about the x -axis. **Answer:** $\frac{\pi^2}{2}$

c) Find the volume of the solid of revolution obtained by rotation of R about the y -axis. **Answer:** $2\pi^2$

d) Find the coordinates of the center of mass of R . **Answer:** $(\frac{\pi}{2}, \frac{\pi}{8})$.

2. Solve the following differential equations:

a) $y' = y$. **Answer:** $y = Ae^x$

b) $y' = \frac{1}{y}$. **Answer:** $y = \pm\sqrt{2x + C}$

c) $y' = \frac{x}{y}$, $y(0) = 1$. **Answer:** $y = \sqrt{x^2 + 1}$

3. Compute the following integrals:

a) $\int \frac{3x+5}{x^2+2x} dx$ **Answer:** $\frac{1}{2} \ln|x+2| + \frac{5}{2} \ln|x| + C$

b) $\int \frac{3x+5}{x^2+2x+2} dx$ **Answer:** $\frac{3}{2} \ln|x^2+2x+2| + 2 \arctan(x+1) + C$

c) $\int \sin(3x) \cos(5x) dx$ **Answer:** $\frac{1}{2} (-\frac{1}{8} \cos(8x) + \frac{1}{2} \cos(2x)) + C$

d) $\int x\sqrt{4-x^2} dx$ **Answer:** $-\frac{1}{3}(4-x^2)^{\frac{3}{2}} + C$

e) $\int x \ln x dx$ **Answer:** $\frac{1}{2}x^2 \ln(x) - \frac{x^2}{4} + C$

4. Compute the following definite integrals:

a) $\int_2^3 \frac{3x+5}{x^2+2x+1} dx$ **Answer:** $6 \ln(2) - \frac{1}{2} - (3 \ln(3) - \frac{2}{3}) = \ln(\frac{64}{27}) + \frac{1}{6}$

b) $\int_0^\pi \sin^3 x \cos^5 x dx$ **Answer:** 0

c) $\int_2^3 \frac{dx}{x \ln^2 x}$ **Answer:** $-\frac{1}{\ln(3)} + \frac{1}{\ln(2)}$

d) $\int_0^1 \frac{x dx}{\sqrt{4-x^2}}$ **Answer:** $-\sqrt{3} + 2$

e) $\int_0^\pi x \sin x dx$. **Answer:** π

5. Find the lengths of the following curves:

a) $y = \frac{1}{2}x^2$, $1 \leq x \leq 2$. **Answer:** $\sqrt{5} + \frac{\ln(2+\sqrt{5})}{2} - \frac{\sqrt{2+\ln(1+\sqrt{2})}}{2}$

b)* $x = 3 \cos t + \cos(3t)$, $y = 3 \sin t + \sin(3t)$, $0 \leq t \leq 2\pi$. **Answer:** 24

6. A gate in a reservoir has a shape of a half-circle of radius R with the center at water level. Find the total force of water pressure on this gate, if the water density equals ρ and the gravity acceleration equals g .

Answer: $\frac{2\rho g R^3}{3}$.

7. Determine the surface area of the solid obtained by rotating $y = x^3, 1 \leq x \leq 2$, about the x -axis.

Answer: $2\pi \left(\frac{145\sqrt{145}}{54} - \frac{5\sqrt{10}}{27} \right)$

8**. Compute the integral $\int \frac{dx}{(x+1)(x+2)(x+3)}$.

Answer: $-\ln|x+2| + \frac{1}{2}\ln|x+3| + \frac{1}{2}\ln|x+1| + C$