# MAT 21B, Spring 2019
# Solutions to homework 2

**Section 5.4:** 12. (10 points) Compute the integral

$$\int_0^{\pi/3} 4\frac{\sin(u)}{\cos^2(u)}du.$$

**Solution 1:** Let $w = \cos(u)$, then $dw = -\sin(u)du$. If $u = 0$ then $w = \cos(u) = 1$, if $u = \pi/3$ then $w = \cos(\pi/3) = 1/2$. Therefore

$$\int_0^{\pi/3} 4\frac{\sin(u)}{\cos^2(u)}du = 4\int_1^{1/2} \frac{-dw}{w^2} = -4\int_1^{1/2} w^{-2}dw =$$

$$\frac{4}{w}|_1^{1/2} = \frac{4}{1/2} - \frac{4}{1} = 8 - 4 = 4.$$

**Solution 2:**

$$\int \frac{\sin(u)}{\cos^2(u)}du = \sec(u) + C,$$

so

$$\int_0^{\pi/3} 4\frac{\sin(u)}{\cos^2(u)}du = 4\sec(u)|_0^{\pi/3} = \frac{4}{\cos(u)}|_0^{\pi/3} = \frac{4}{1/2} - \frac{4}{1} = 8 - 4 = 4.$$

20. (10 points) $\int_{-\sqrt{3}}^{\sqrt{3}}(t+1)(t^2+4)dt$.

**Solution:** We have $(t+1)(t^2+4) = t^3 + t^2 + 4t + 4$, so

$$\int_{-\sqrt{3}}^{\sqrt{3}}(t+1)(t^2+4)dt = \int_{-\sqrt{3}}^{\sqrt{3}}(t^3+t^2+4t+4)dt =$$

$$(\frac{t^4}{4}+\frac{t^3}{3}+2t^2+4t)|_{-\sqrt{3}}^{\sqrt{3}} = \left(\frac{9}{4}+\frac{3\sqrt{3}}{3}+6+4\sqrt{3}\right) - \left(\frac{9}{4}-\frac{3\sqrt{3}}{3}+6-4\sqrt{3}\right) =$$

$$2(\frac{3\sqrt{3}}{3}+4\sqrt{3}) = 10\sqrt{3}.$$

30. (10 points) $\int_1^2(\frac{1}{x}-e^{-x})dx$.

**Solution:**

$$\int_1^2(\frac{1}{x}-e^{-x})dx = (\ln|x|+e^{-x})|_1^2 = \ln(2)+e^{-2}-ln(1)-e^{-1} = \ln(2)+e^{-2}-e^{-1}.$$