MAT 21B, Spring 2019
Solutions to homework 5

Section 6.2: 6. (10 points) Find the volume of the solid obtained by rotation of the region below the graph

$$f(x) = \frac{9x}{\sqrt{x^3+9}},\ 0 \le x \le 3$$

around the y-axis.

Solution: We have

$$V = 2\pi \int_0^3 xf(x)dx = 2\pi \int_0^3 \frac{9x^2}{\sqrt{x^3+9}}dx.$$

Let $u = x^3 + 9$, then $du = 3x^2$ and

$$V = 2\pi \int_9^{36} \frac{3du}{\sqrt{u}} = 6\pi \int_9^{36} u^{-1/2}du = 6\pi \cdot (2u^{1/2})|_9^{36} = 12\pi(6-3) = 36\pi.$$

Section 6.4: 27. (10 points) Your company plans to coat 5000 woks inside and outside with 0.5mm think enamel. Each wok is a part of the surface of the sphere of radius 16 between $x = -7$ and $x = -16$. How much enamel do you need?

Solution: Let us find the area of the surface first. We have

$$S = 2\pi \int_7^{16} f(x)\sqrt{1+f'(x)^2}dx,$$

where $f(x) = \sqrt{256 - x^2}$, so

$$f'(x) = \frac{-2x}{2\sqrt{256-x^2}} = \frac{-x}{\sqrt{256-x^2}},$$

$$(f'(x))^2 = \frac{x^2}{256-x^2},$$

so

$$1 + (f'(x))^2 = 1 + \frac{x^2}{256-x^2} = \frac{256 - x^2 + x^2}{256-x^2} = \frac{256}{256-x^2},$$

$$\sqrt{1+(f'(x))^2} = \sqrt{\frac{256}{256-x^2}} = \frac{16}{\sqrt{256-x^2}},$$

and finally

$$f(x)\sqrt{1+(f'(x))^2} = \sqrt{256-x^2}\cdot\frac{16}{\sqrt{256-x^2}} = 16.$$

Therefore

$$S = 2\pi\int_7^{16} 16dx = 32\pi x|_7^{16} = 32\pi(16-7) = 32\pi * 9 \approx 905cm^2.$$

Now for one wok we need to cover it with thickness 0.5mm=0.05cm, so we need

$$905\cdot 0.05 \approx 45cm^3$$

of enamel. For 5000 woks we then need about 226,194cm^3 $\approx$ 226 liters of enamel of each color.

32. (10 points) Find the area of the surface generated by the rotation about the x-axis the graph of $y = (1-x^{2/3})^{3/2}$ for $-1 \le x \le 1$.

Solution: We can restrict to $0 \le x \le 1$ and double the result. We have

$$S = 2\pi\int_0^1 f(x)\sqrt{1+f'(x)^2}dx,$$

so

$$f(x) = (1-x^{2/3})^{3/2},\ f'(x) = \frac{3}{2}(1-x^{2/3})^{1/2}\cdot\frac{2}{3}(-x^{-1/3}) = -x^{-1/3}(1-x^{2/3})^{1/2}.$$

Now

$$(f'(x))^2 = x^{-2/3}(1-x^{2/3}) = x^{-2/3}-1,\ 1+(f'(x))^2 = x^{-2/3},$$

and

$$\sqrt{1+(f'(x))^2} = x^{-1/3}.$$

We have

$$S = 2\pi\int_0^1 f(x)\sqrt{1+f'(x)^2}dx = 2\pi\int_0^1(1-x^{2/3})^{3/2}\cdot x^{-1/3}dx.$$

Let $u = 1-x^{2/3}$, then $du = -\frac{2}{3}x^{-1/3}dx$, so $x^{-1/3}dx = -\frac{3}{2}du$. We have

$$S = 2\pi\int_1^0 u^{3/2}(-\frac{3}{2}du) = 6\pi\int_0^1 u^{3/2}du = 3\pi\cdot\frac{2}{5}u^{5/2}|_0^1 = 3\pi\cdot\frac{2}{5} = \frac{6\pi}{5}.$$

Now we double the answer and get total area $\frac{12\pi}{5}$.