Section 6.2: 6. (10 points) Find the volume of the solid obtained by rotation of the region below the graph
\[ f(x) = \frac{9x}{\sqrt{x^3 + 9}}, \quad 0 \leq x \leq 3 \]
around the \( y \)-axis.

Solution: We have
\[ V = 2\pi \int_0^3 xf(x)dx = 2\pi \int_0^3 \frac{9x^2}{\sqrt{x^3 + 9}}dx. \]
Let \( u = x^3 + 9 \), then \( du = 3x^2 \) and
\[ V = 2\pi \int_9^{36} \frac{3du}{\sqrt{u}} = 6\pi \int_9^{36} u^{-1/2}du = 6\pi \cdot (2u^{1/2})|_9^{36} = 12\pi(6 - 3) = 36\pi. \]

Section 6.4: 27. (10 points) Your company plans to coat 5000 woks inside and outside with 0.5mm thick enamel. Each wok is a part of the surface of the sphere of radius 16 between \( x = -7 \) and \( x = -16 \). How much enamel do you need?

Solution: Let us find the area of the surface first. We have
\[ S = 2\pi \int_{-7}^{-16} f(x)\sqrt{1 + f'(x)^2}dx, \]
where \( f(x) = \sqrt{256 - x^2} \), so
\[ f'(x) = \frac{-2x}{2\sqrt{256 - x^2}} = \frac{-x}{\sqrt{256 - x^2}}, \]
\[ (f'(x))^2 = \frac{x^2}{256 - x^2}, \]
so
\[ 1 + (f'(x))^2 = 1 + \frac{x^2}{256 - x^2} = \frac{256 - x^2 + x^2}{256 - x^2} = \frac{256}{256 - x^2}, \]
\[
\sqrt{1 + (f'(x))^2} = \sqrt{\frac{256}{256 - x^2}} = \frac{16}{\sqrt{256 - x^2}},
\]
and finally
\[
f(x)\sqrt{1 + (f'(x))^2} = \sqrt{256 - x^2} \cdot \frac{16}{\sqrt{256 - x^2}} = 16.
\]
Therefore
\[
S = 2\pi \int_7^{16} 16 \, dx = 32\pi x |_{7}^{16} = 32\pi (16 - 7) = 32\pi \cdot 9 \approx 905cm^2.
\]
Now for one wok we need to cover it with thickness 0.5mm=0.05cm, so we need
\[
905 \cdot 0.05 \approx 45cm^3
\]
of enamel. For 5000 woks we then need about 226,194cm^3 \approx 226 liters of enamel of each color.

32. (10 points) Find the area of the surface generated by the rotation about the x-axis the graph of \(y = (1 - x^{2/3})^{3/2}\) for \(-1 \leq x \leq 1\).

**Solution:** We can restrict to \(0 \leq x \leq 1\) and double the result. We have
\[
S = 2\pi \int_0^1 f(x) \sqrt{1 + f'(x)^2} \, dx,
\]
so
\[
f(x) = (1 - x^{2/3})^{3/2}, \quad f'(x) = \frac{3}{2}(1 - x^{2/3})^{1/2} \cdot \frac{2}{3}(-x^{-1/3}) = -x^{-1/3}(1 - x^{2/3})^{1/2}.
\]
Now
\[
(f'(x))^2 = x^{-2/3}(1 - x^{2/3}) = x^{-2/3} - 1, \quad 1 + (f'(x))^2 = x^{-2/3},
\]
and
\[
\sqrt{1 + (f'(x))^2} = x^{-1/3}.
\]
We have
\[
S = 2\pi \int_0^1 f(x) \sqrt{1 + f'(x)^2} \, dx = 2\pi \int_0^1 (1 - x^{2/3})^{3/2} \cdot x^{-1/3} \, dx.
\]
Let \(u = 1 - x^{2/3},\) then \(du = -\frac{3}{2}x^{-1/3} \, dx,\) so \(x^{-1/3} \, dx = -\frac{3}{2} \, du.\) We have
\[
S = 2\pi \int_0^1 u^{3/2} (-\frac{3}{2} \, du) = 6\pi \int_0^1 u^{3/2} \, du = 3\pi \cdot \frac{2}{5} u^{5/2} |_0^1 = 3\pi \cdot \frac{2}{5} = \frac{6\pi}{5}.
\]
Now we double the answer and get total area \(\frac{12\pi}{5}.\)