MAT 21B, Spring 2019
Solutions to homework 7

Written Assignment:

Section 8.2: 20. (10 points) Compute $\int t^2 e^{4t} dt$.

Solution: Let us integrate by parts: $t^2 = u$, $e^{4t}dt = dv$, so $v = \frac{1}{4}e^{4t}$. Therefore

$$\int t^2 e^{4t} dt = t^2 \cdot \frac{1}{4}e^{4t} - \frac{1}{4}\int e^{4t} d(t^2) = \frac{1}{4}t^2 e^{4t} - \frac{1}{2}\int t e^{4t} dt.$$

We can integrate by parts one more time:

$$\int t e^{4t} dt = \frac{1}{4}\int t d(e^{4t}) = \frac{1}{4}te^{4t} - \frac{1}{4}\int e^{4t} dt = \frac{1}{4}te^{4t} - \frac{1}{16}e^{4t}.$$

By combining these answers, we get

$$\int t^2 e^{4t} dt = \frac{1}{4}t^2 e^{4t} - \frac{1}{2}\left(\frac{1}{4}te^{4t} - \frac{1}{16}e^{4t}\right) =$$

$$\frac{1}{4}t^2 e^{4t} - \frac{1}{8}te^{4t} + \frac{1}{32}e^{4t} + C.$$

Section 8.3: 54. (10 points) $\int_0^{\pi/2} \sin x \cos x dx$.

Solution: Let $u = \sin x$, then $du = \cos x dx$ and

$$\int \sin x \cos x dx = \int u du = \frac{1}{2}u^2 + C = \frac{1}{2}\sin^2(x) + C.$$

Therefore

$$\int_0^{\pi/2} \sin x \cos x dx = \frac{1}{2}\sin^2(x)|_0^{\pi/2} = 1/2 - 0 = 1/2.$$

Section 8.5: 12. (10 points) $\int \frac{2x+1}{x^2-7x+12} dx$

Solution: We can factor $x^2 - 7x + 12 = (x-3)(x-4)$ and write the partial fraction decomposition:

$$\frac{2x+1}{x^2-7x+12} = \frac{A}{x-3} + \frac{B}{x-4}.$$

By cross-multiplying, we get

$$2x + 1 = A(x - 4) + B(x - 3).$$

At $x = 3$ we get $7 = A(-1)$, so $A = -7$, at $x = 4$ we get $9 = B(1)$, so $B = 9$. We get

$$\int \frac{2x+1}{x^2 - 7x + 12} dx = -7 \int \frac{dx}{x-3} + 9 \int \frac{dx}{x-4} = -7 \ln|x-3| + 9 \ln|x-4| + C.$$